

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Fifth Semester)

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

ADVANCED DIFFERENTIAL EQUATIONS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions
ALL questions carry EQUAL marks (5 x 1 = 5)

- 1 Find the eigen values of the matrix $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$.
(i) 3,-3 (ii) -5,3
(iii) -3,-5 (iv) 3,5
- 2 The radius of convergence of the series $\sum_{n=1}^{\infty} n^n z^n$ is -----
(i) 0 (ii) ∞
(iii) 1 (iv) e
- 3 $\Gamma(1/2) =$ -----
(i) π (ii) $\pi/2$
(iii) $\sqrt{\pi}$ (iv) $\sqrt{\pi}/2$
- 4 A critical point is called -----if some neighbourhood of it contains no other critical path.
(i) origin (ii) proper node
(iii) isolated (iv) improper node
- 5 Quasi-linear equation is also called a -----equation.
(i) uniform nonlinear (ii) Charpit's
(iii) Monge (iv) non-uniform nonlinear

SECTION - B (15 Marks)

Answer ALL Questions
ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6 a Find a general solution of the system $x' = \begin{bmatrix} 1 & -3 \\ 3 & 7 \end{bmatrix} x$.
OR
b Let $A = \begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix}$. Find AB and BA.
- 7 a Solve the equation $x^2 y' = y - x - 1$.
OR
b Investigate the nature of the point $x = 0$ for the differential equation $x^4 y'' + (x^2 \sin x) y' + (1 - \cos x) = 0$.
- 8 a Solve the equation $4x^2 y'' + 8xy' + (x^4 - 3)y = 0$.
OR
b Evaluate $\int x^2 J_0(x) dx$.
- 9 a Determine the type and stability of the critical point (0,0) of the almost linear system $\frac{dx}{dt} = 4x + 2y + 2x^2 - 3y^2$, $\frac{dy}{dt} = 4x - 3y + 7xy$.
OR
b Explain critical points of Linear system.

Cont...

- 10 a Find a complete integral of the equation $(p + q)(z - xp - yq) = 1$.
OR
b Solve the equation $q^2r - 2pqs + p^2t = 0$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11 a Solve the initial value problem $\frac{dx}{dt} = \begin{bmatrix} 3 & -2 & 0 \\ -1 & 3 & -2 \\ 0 & -1 & 3 \end{bmatrix} x(0) = \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix}$.
OR
b Find a general solution of the system $x_1' = 4x_1 + 2x_2, x_2' = 3x_1 - x_2$.
- 12 a Solve the equation $y' + 2y = 0$.
OR
b Find the Frobenius series of $2x^2y'' + 3xy' - (x^2 + 1)y = 0$.
- 13 a Determine whether or not the equation $x^2y'' - xy' + (x^2 - 8)y = 0$ has two linearly independent Frobenius series solutions.
OR
b Explain Bessel function identities with an example.
- 14 a Find all critical points of the given system, and investigate the type and stability of each. $\frac{dx}{dt} = x - y; \frac{dy}{dt} = x^2 - y$.
OR
b Find the critical points of the system
 $\frac{dx}{dt} = 14x - 2x^2 - xy; \frac{dy}{dt} = 16y - 2y^2 - xy$.
- 15 a Solve the equation $r + 4s + t + rt - s^2 = 2$.
OR
b Find a complete integral of the equation: $p^2x + q^2y = z$.

Z-Z-Z

END