23MCU312

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Third Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATION**

DISCRETE MATHEMATICS AND GRAPH THEORY

Time: Three Hours   Maximum: 50 Marks

SECTION-A (5 Marks)
Answer ALL questions
ALL questions carry EQUAL marks   (5 x 1 = 5)

1. Which of the following is called as Idempotent law
    - (i) $p \lor p \Leftrightarrow p$
    - (ii) $p \lor F_0 \Leftrightarrow p$
    - (iii) $p \lor q \Leftrightarrow q \lor p$
    - (iv) $\neg \neg p \Leftrightarrow p$

2. Simplification of the expression $x + xy$ is
    - (i) y
    - (ii) x
    - (iii) xy
    - (iv) $x + y$

3. Which of the following is called as Golden ratio?
    - (i) $\frac{1+\sqrt{5}}{2}$
    - (ii) $\frac{1-\sqrt{5}}{2}$
    - (iii) $1 + \frac{\sqrt{5}}{2}$
    - (iv) $1 - \frac{\sqrt{5}}{2}$

4. A vertex is called pendant vertex if its
    - (i) degree 0
    - (ii) degree 1
    - (iii) degree 2
    - (iv) degree 3

5. In a binary root tree, for each vertex v,
    - (i) $od\ (v) = 0,1\ or\ 2$
    - (ii) $od\ (v) = 0\ or\ 1$
    - (iii) $od\ (v) = 1\ or\ 2$
    - (iv) $od\ (v) = 0$

SECTION - B (15 Marks)
Answer ALL Questions
ALL Questions Carry EQUAL Marks   (5 x 3 = 15)

6. (a) Develop the truth table for the compound statement $p \lor (q \land r)$.

    (OR)

    (b) Simplify the expression $wx + \overline{\overline{x}z} + (y + \overline{z})$ where w, x, y, z are Boolean variables.

7. (a) Bring out the coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + \cdots)^4$.

    (OR)

    (b) Show that for all $n \in Z^+$, $\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2$.

8. (a) Solve the recurrence relation $a_n = 7a_{n-1}$, where $n \geq 1$ and $a_2 = 98$.

    (OR)

    (b) Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200$, where $n \geq 0$ and $a_0 = 3000,\ a_1 = 3300$.

9. (a) Let $G = (V, E)$ be a loop free connected planar graph with $|V| = v, |E| = e > 2, r$ regions. Show that $3r \leq 2e$

(OR)

(b) For each graph G, Show that the constant term in $P(G, \lambda)$ is 0.

10. (a) If a, b are distinct vertices in a tree $T = (V, E)$, then prove that there is a unique path that connects these vertices.

(OR)

(b) Explain the MergeSort Algorithm.

SECTION -C (30 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 x 6 = 30)

11. (a) Discover the disjunctive normal form for $g(w, x, y, z) = wx\bar{y} + wy\bar{z} + xy$

(OR)

(b) Prove that $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$.

12. (a) Identify the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$

(OR)

(b) Construct a formula to express $0^2 + 1^2 + 2^2 + \cdots + n^2$ as a function of n.

13. (a) Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n-1} - a_n$, where $n \geq 0$ and $a_0 = 0$, $a_1 = 1$, $a_2 = 2$.

(OR)

(b) Solve the recurrence relation $a_n - 3a_{n-1} = n$, where $n \geq 1$ and $a_0 = 1$.

14. (a) Let $G = (V, E)$ be an undirected graph or multigraph with no isolated vertices. Then prove that G has Euler circuit if and only if G is connected and every vertex in G has even degree.

(OR)

(b) State and Prove Decomposition Theorem for Chromatic Polynomials.

15. (a) In every tree $T = (V, E), |V| = |E| + 1$. Justify this statement by proving it.

(OR)

(b) Let $T = (V, E)$ be a complete m - ary tree of height h with l leaves. Then show that $l \leq m^h$ and $h \geq [log_m l]$.

Z-Z-Z   END