22MCU102N/ 22MCU102

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Newton's Law of cooling is given by _____ a) $\frac{dT}{dt} = k(T-A)$ b) $\frac{dT}{dt} = -k(T-A)$ c) $\frac{dt}{dT} = -k(T-A)$ d) $\frac{dt}{dT} = k(T-A)$	K1	CO1
	2	A linear first-order differential equation has ____ singular solutions. a) no b) 1 c) 'n' d) many	K1	CO1
2	3	The general solution of $y''+y'=0$ is _____ a) $A\cos x - B\sin x$ b) $A\sin x + B\cos x$ c) $A\sin x - B\cos x$ d) $A\cos x + B\sin x$	K1	CO2
	4	The particular solution of $y''+4y=12x$ is _____ a) x b) 12x c) 3x d) 0	K2	CO2
3	5	The fifth-order differential equation $9y^{(5)} - 6y^{(4)} + y(3) = 0$ has the triple root r = ___ and the double root r = _____ a) $0, \frac{1}{3}$ b) $\frac{1}{3}, 0$ c) 0, 1 d) 1, 0	K2	CO3
	6	The basic circuit equation is _____ a) $L\frac{dI}{dt}+RI+\frac{1}{C}Q=E(t)$ b) $L\frac{dI}{dt}-RI+\frac{1}{C}Q=E(t)$ c) $L\frac{dI}{dt}-RI-\frac{1}{C}Q=E(t)$ d) $L\frac{dI}{dt}+RI+\frac{1}{C}Q+E(t)=0$	K1	CO3
4	7	Laplace transform of function f(t) is given by _____ a) $F(t)=\int_0^\infty e^{-t}f(t)dt$ b) $F(s)=\int_0^\infty e^{-st}f(t)dt$ c) $f(t)=\int_0^\infty e^{-t}f(t)dt$ d) $f(s)=\int_0^\infty e^{-st}f(t)dt$	K2	CO4
	8	$L(t^{-½}) =$ _____ a) $\sqrt{\frac{\pi}{s^2}}$ b) $\frac{\pi}{s}$ c) $\sqrt{\frac{\pi}{s^2}}$ d) $\sqrt{\frac{\pi}{s}}$	K2	CO4

Cont...

5	9	The convolution process associated with the Laplace Transform in time domain results into_____ a) Simple multiplication in complex frequency domain b) Simple division in complex frequency domain c) Simple multiplication in complex time domain d) Simple division in complex time domain	K1	CO5
	10	Which of the following is not a property of Laplace transforms? a) Linearity b) Differentiation property c) Convolution property d) Time-shift property	K2	CO5

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks $(5 \times 7 = 35)$

Module No.	Question No.	Question	K Level	CO
1	11.a.	A lunar lander is falling freely toward the surface of the moon at a speed of 450 m/s. Its retrorockets, when fired, provide a constant deceleration of $2.5 m/s^2$. At what height above the lunar surface should the retrorockets be activated to ensure a "soft touchdown" (v = 0 at impact)?	K2	CO1
		(OR)		
	11.b.	Solve the differential equation $\frac{dy}{dx} = (x+y+3)^2$.		
2	12.a.	Evaluate the initial value problem $y'' + 2y' + y = 0$ given $y(0) = 5, y'(0) = -3$	K3	CO2
		(OR)		
	12.b.	If $y_1, y_2, \ldots, y_n$ are n solutions of the homogeneous nth-order linear equation on an open interval I and $W = W(y_1, y_2, \ldots, y_n)$, then demonstrate that (a) If $y_1, y_2, \ldots, y_n$ are linearly dependent, then $W = 0$ on I. (b) If $y_1, y_2, \ldots, y_n$ are linearly independent, then $W \neq 0$ at each point of I.		
3	13.a.	Find the general solution of $y^{(4)} + 4y = 0$.	K2	CO3
		(OR)		
	13.b.	Find the particular solution of $3y'' + y' - 2y = 2\cos x$.		
4	14.a.	Evaluate L(t sin 6t).	K3	CO4
		(OR)		
	14.b.	Consider a mass-and-spring system with m = ½, k = 17 and c = 3 in mks units. If x(t) is the displacement of the mass m from its equilibrium position and the mass is set in motion with x(0) = 3 and x'(0) = 1, then evaluate x(t) for the resulting damped free oscillations.		
5	15.a.	Find $L^{-1}(\tan^{-1}\left(\frac{1}{s}\right))$.	K2	CO5
		(OR)		
	15.b.	Compute the inverse Laplace transform of $Y(s) = \frac{1}{3-4s} + \frac{3-2s}{s^2+49}$		

SECTION -C (30 Marks)

Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Suppose that at time t = 0, 10 thousand people in a city with population M = 100 thousand people have heard a certain rumor. After 1 week the number P(t) of those who have heard, it has increased to P(l) = 20 thousand. Assuming that P(t) satisfies a logistic equation, when will 80% of the city's population have heard the rumor?	K4	CO1
2	17	If y_1 and y_2 are two linearly independent solutions of the homogeneous equation $y'' + p(x)y' + q(x)y = 0$ with p and q continuous on the open interval I and if Y is any solution, then prove that there exists numbers c_1 and c_2 such that $Y(x) = c_1y_1(x) + c_2y_2(x)$.	K3	CO2
3	18	Solve the initial value problem $y'' - 3y' + 2y = 3e^{-x} - 10 \cos 3x$; $y(0) = 1$ and $y'(0) = 2$.	K3	CO3
4	19	Using Laplace Transform, solve the initial value problem $x'' - x' - 6x = 0$; $x(0) = 2$, $x'(0) = -1$.	K4	CO4
5	20	A mass that weighs 32 lb (mass m = 1 slug) is attached to the free end of a long, light spring that is stretched 1 ft by a force of 4 lb (k = 4 lb/ft). The mass is initially at rest in its equilibrium position. Beginning at time t = 0 (seconds), an external force f(t) = cos 2t is applied to the mass, but at time $t = 2\pi$ this force is turned off (abruptly discontinued) and the mass is allowed to continue its motion unimpeded. Determine the resulting position function x (t) of the mass.	K4	CO5

Z-Z-Z END