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20MAP22A

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2022
(Fourth Semester)

Branch – **MATHEMATICS**

DISCIPLINE SPECIFIC ELECTIVE – II: COMPUTATIONAL METHODS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1. Which one of following is Iterative Methods?
   (i) Gauss elimination. (ii) Gauss-Jordan method
   (iii) Gauss seidal method (iv) Jordan method

2. ______________ gives the largest eigen value in magnitude.
   (i) Power method (ii) Gauss-Seidel method
   (iii) Gauss-Jacobi method (iv) Jacobi method

3. $\int_{-1}^{1} f(x)dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$ is ______________.
   (i) Gauss-Legendre three point rule (ii) One point Gauss formula
   (iii) Gauss-Chebychev (iv) Two point Gauss rule

4. The error term in the Gauss three point rule is ______________.
   (i) $R(f) = \frac{1}{15750} f^{(6)}(\xi), -1 < \xi < 1.$ (ii) $R(f) = \frac{1}{3} f''(\xi), -1 < \xi < 1$
   (iii) $R(f) = \frac{1}{15750} f^{(5)}(\xi), -1 < \xi < 1$ (iv)) $R(f) = \frac{1}{15750} f^{(4)}(\xi), -1 < \xi < 1$

5. All predictor methods are ______________.
   (i) Implicit (ii) Direct (iii) indirect (iv) direct

6. $y_{i+1}^{(p)} = y_{i-3} + \frac{4h}{3}[2f_i - f_{i-1} + 2f_{i-2}]$ formula is known as ______________.
   (i) Milne's predictor (ii) Adams-Bashforth
   (iii) Milne-Simpson's method (iv) Adams-Bashforth-Moulton

7. The partial differential equation is called a Elliptic equation if it satisfies ______________.
   (i) $B^2 - 4AC < 0$ (ii) $B^2 - 4AC > 0$ (iii) $B^2 - 4AC = 0$ (iv) $B - AC = 0$

8. Two dimensions equation $u_{xx} + u_{yy} = 0$ is called ______________.
   (i) Laplace equation (ii) Poisson equation (iii) Liebmann method (iv) Gauss-Jacobi

9. The order and truncation error of the Schmidt method is ______________.
   (i) $O(k + h^2)$ (ii) $O(k^2 - h^2)$ (iii) $O(k^2 + h^4)$ (iv) $O(k^4 + h^4)$

10. Do you recognize the system of equations that is obtained if we apply the Crank Nicolson method?
    (i) tri-diagonal system (ii) Bi conditional (iii) Upper system (iv) Lower system

SECTION - B (35 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 7 = 35)

11. a) Determine the numerically largest eigen value and the corresponding eigen vector of the following matrix, using the power method $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$

(OR)

b) Solve the system of equations $26x_1 + 2x_2 + 2x_3 = 12.6, 3x_1 + 27x_2 + x_3 = -14.3,$ $2x_1 + 3x_2 + 17x_3 = 6.0$ using the Jacobi iteration method.

12. a) Evaluate the integral $\int_1^2 \frac{2x}{1+x^4}$, using Gauss one point rules.

(OR)

b) Evaluate the integral $\int_0^2 \frac{(x^2+2x+1)}{1+(1+x)^4} dx$, by Gauss three point formula.

Cont...

13. a) Solve $y' = 1 - y$ with the initial condition $x = 0, y = 0$, using Euler's algorithm and tabulate the solutions at $x = 0.1, 0.2, 0.3, 0.4$.

(OR)

b). Using the Runge-Kutta method of order 4, find y for $x = 0.1, 0.2, 0.3$ given that $y' = xy + y^2, y(0) = 1$

14. a) Solve the boundary value problem $xy'' + y = 0, y(1) = 1, y(2) = 2$ by second order finite difference method with h = 0.25.

(OR)

b) Solve $u_{xx} + u_{yy} = 0$ numerically under the boundary conditions

$$u(x,0) = 2x, u(0,y) = -y,$$
$$u(x,1) = 2x - 1, u(1,y) = 2 - y$$

with square mesh of width h = 1/3.

15.a) Solve $u_{xx} = 32\,ut$, $0 \le x \le 1$, taking h = 0.5 and $u(x,0) = 0, 0 \le x \le 1$, $u(0,t) = 0, u(1,t) = t, t > 0$. Use an explicit method with $\lambda = 1/2$. Compute for four time steps.

(OR)

b) Solve by Crank-Nicolson method the equation $u_{xx} = u_t$ subject to $u(x,0) = 0, u(0,t) = 0$ and $u(1,t) = t$, for two time steps.

SECTION - C (30 Marks)

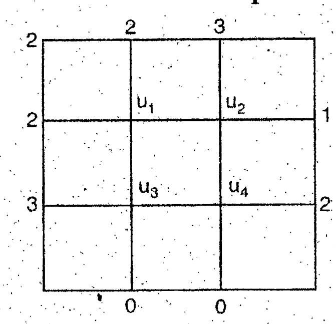
Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16. Find the solution of the system of equations
$45x_1 + 2x_2 + 3x_3 = 58, -3x_1 + 22x_2 + 2x_3 = 47, 5x_1 + x_2 + 20x_3 = 67$
correct to three decimal places, using the Gauss-Seidel iteration method.

17. Evaluate the integral $I = \int_{y=1}^{1.5} \int_{x=1}^{2} \frac{dx\,dy}{x+y}$ using the Simpson's rule with h = 0.5 along x-axis and k = 0.25 along y-axis. The exact value of the integral is I = 0.184401. Find the absolute error in the solution obtained by the Simpson's rule.

18. Given $y' = x^3 + y, y(0) = 2$, the values $y(0.2) = 2.073, y(0.4) = 2.452$, and $y(0.6) = 3.023$ are got by Runge-Kutta method of fourth order. Find y(0.8) by Milne's predictor-corrector method taking $h = 0.2$.

19. Solve $u_{xx} + u_{yy} = 0$ numerically, using five point formula and Liebmann iteration, for the following mesh with uniform spacing and with boundary conditions as shown below in the figure. Obtain the results correct to two decimal places.



20. Solve $u_{xx} = u_{tt}, 0 < x < 1, t > 0$, given $u(x,0) = 0, u_t(x,0) = 0, u(0,t) = 0$ and $u(1,t) = 100 \sin\pi t$. Compute for four time steps with h = 0.25.

Z-Z-Z END