

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Fifth Semester)

Branch – STATISTICS

STATISTICAL INFERENCE - II

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

- 1 The hypothesis under test is
(i) Simple hypothesis (ii) Alternative hypothesis
(iii) Null hypothesis (iv) Composite hypothesis
- 2 If the likelihood ratio is λ , the variable $-2\log \lambda$ is approximately distributed as _____
(i) t – test (ii) F – test (iii) Chi – square test (iv) Z -test
- 3 The range of statistic t is
(i) - 1 to + 1 (ii) - ∞ to ∞ (iii) 0 to ∞ (iv) 0 to 1
- 4 Degrees of freedom for statistic χ^2 in case of contingency table of order(2 x 2) is
(i) 3 (ii) 2 (iii) 1 (iv) 4
- 5 In case of two attributes A and B, the class frequency (αB) in terms of other class frequencies is
(i) (B) + (AB) (ii) (B) - (AB) (iii) (Ab) – (B) (iv) N – (AB)

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6 a Let $X \sim N(\mu, 4)$, μ known. To test $H_0 : \mu = -1$ against $H_1 : \mu = 1$, based on a sample size 10 from this population, we use the critical region: $x_1 + 2x_2 + \dots + 10x_{10} \geq 0$. What is its size? What is the power of the test?
OR
b Let X have a p.d.f of the form $f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & 0 < x < \infty, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$
To test $H_0 : \theta = 2$, against $H_1 : \theta = 1$, use the random sample x_1, x_2 of size 2 and define a critical region $W = \{(x_1, x_2) : 9.5 \leq x_1 + x_2\}$ Find : (i) Power of the test (ii) Significance level of the test.
- 7 a Mention the properties of likelihood ratio test.
OR
b If λ is the likelihood ratio for testing a simple hypothesis H_0 and if $CU = \phi(\lambda)$ is a monotonic increasing function of λ then the test based on U is equivalent to the likelihood ratio test. Prove that the critical region for the test based on U is $\phi(0) < U < \phi(\lambda_0)$, $[\phi(\lambda_0) < U < \phi(0)]$
- 8 a The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased in 153.7 and showed a standard deviation of 17.2. was the advertising campaign successful?
OR
b A test was given to a large group of boys who scored on the average 64.5 marks. The same test was given to a group of 400 boys who scored an average of 62.5 marks with a standard deviation 12.5 marks. Examine if the difference is significant.
- 9 a List the applications of 'F' distribution.
OR
b It is believed that the precision of an instrument is no more than 0.16. Write down the null and alternate hypothesis for testing this belief. Carry out the test at 1% level given 11 measurements of the same subject on the instrument: 2.5, 2.3, 2.4, 2.3, 2.5, 2.7, 2.5, 2.6, 2.6, 2.7, 2.5

- 10 a 800 candidates of both sexes appeared at an examination. The boys outnumbered the girls by 15 % of the total. The number of candidates who passed exceed the number failed by 480. Equal number of boys and girls failed in the examination, prepare a 2 x 2 table and find the coefficient association.

OR

- b a Show that if $\frac{(A)}{N} = x, \frac{(B)}{N} = 2x, \frac{(C)}{N} = 3x,$ and $\frac{(AB)}{N} = \frac{(BC)}{N} = \frac{(CA)}{N} = y$ Then the value of neither x nor y can exceed $\frac{1}{4}$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11 a State and prove Neyman Pearson Lemma.

OR

- b Given a random sample x_1, x_2, \dots, x_n from the distribution with p.d.f $f(x, \theta) = \theta e^{-\theta x}, x > 0$. show that there exists no UMP test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$

- 12 a Show that the normal distribution with mean zero and variance σ^2 , the best critical region for $H_0: \sigma = \sigma_0$ against the alternative $H_1: \sigma = \sigma_1$ is of the form

$$\sum_{i=1}^n x_i^2 \leq a_\alpha, \text{ for } \sigma_0 > \sigma_1 \quad \text{And} \quad \sum_{i=1}^n x_i^2 \geq b_\alpha, \text{ for } \sigma_0 < \sigma_1$$

Show that the power of the best critical region when $\sigma_0 > \sigma_1$ is $F\left(\frac{\sigma_0^2}{\sigma_1^2}, \chi^2_{\alpha, n}\right)$, where $\chi^2_{\alpha, n}$ is lower 100 α percent point and F is the distribution function of the χ^2 distribution

OR

- b Given a random sample x_1, x_2, \dots, x_n from the distribution with p.d.f $f(x, \theta) = \theta e^{-\theta x}, x > 0$. show that there exists no UMP test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$
- 13 a Two salesman A and B are working in a certain district. From a sample survey conducted by the head office, the following results were obtained. State whether there is any significant difference in the average sales between the two salesmen.

	A	B
No of sales	20	18
Average sales (in Rs)	170	205
Standard deviation (in Rs)	20	25

OR

- b A certain stimulus administered to each of the 12 patients resulted in the following increase of blood pressure: 5,2,8,-1,3,0,-2,1,5,0,4,6. Can it be concluded that the stimulus will, in general, be accompanied by an increase in blood pressure?
- 14 a In a test given to two groups of students drawn from two normal populations, the marks obtained were obtained as follows.

Group A	: 18	20	36	50	49	36	34	49	41
Group B	: 29	28	26	35	30	44	46		

Examine at 5% level, whether the two population have the same variance.

OR

- b The following figures show the distribution of digits in numbers chosen at random from a telephone directory:

Digits	0	1	2	3	4	5	6	7	8	9	Total
Frequency	1026	1107	997	966	1075	933	1107	972	964	853	10000

Test whether the digits may be taken to occur equally frequently in the directory.

- 15 a The following table gives the distribution of students and also of regular players among them, according to age in completed years:

Age in years	15	16	17	18	19	20
No of students	250	200	150	120	100	80
Regular players	200	150	90	48	30	12

Calculate the coefficient of association between majority and playing habit, on the assumption that majority is attained in 18th year.

OR

- b Among the adult population of a certain town 50 percent are males, 60 percent are wage earners and 50m percent are 45 years of age or over, 10 percent of the males are not wage – earners and 40 percent of the males are under 45. Make the best possible inference about the limits with in which the percentage of persons of 45 years or over are wage earners.