22STP101N/22STP101

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024
(First Semester)

Branch - **STATISTICS**

REAL ANALYSIS AND LINEAR ALGEBRA

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	At which point, the function $f(x) = \frac{x^2 - 1}{x - 1}$ not continuous? (a) $x = 1$ (b) $x = 2$ (c) $x = -1$ (d) $x = 0$	K1	CO1
	2	The absolute maximum and minimum of a continuous function on a closed interval [a, b] always exist because ______________. (a) The function is differentiable (b) The function is increasing (c) The interval is closed and bounded (d) The function is linear	K2	CO1
2	3	A sequence whose range is a sub-set of R may be called a ____________. (a) Series (b) Sequence (c) Real Series (d) Real Sequence	K1	CO2
	4	If $a_1 + a_2 + \cdots$ converges to s, then $a_2 + a_3 + \cdots$ is converge to __________ (a) s (b) a_1 (c) s - a_1 (d) s + a_1	K2	CO2
3	5	Which of the following is called first mean value theorem? (a) Every continuous function is differentiable (b) Every differentiable function is continuous (c) If a function is continuous on a closed interval [a,b] and differentiable on the open interval (a,b), then there exists at least one c in (a,b) such that f ′ (c)=0. (d) If f(a) =f(b), then there exists atleast one c in (a,b) such that f ′ (c)=0.	K1	CO3
	6	If f is a bounded function on the closed bounded interval [a,b], we say that f is Riemann integrable on [a,b] if ____ (a) $\int_0^b f = \int_a^b f$ (b) $\int_{\underline{a}}^{\overline{b}} f = \int_{\underline{a}}^{\overline{b}} f$ (c) $\int_{\underline{a}}^{b} f = \int_a^{\overline{b}} f$ (d) $\int_{\underline{a}}^{\overline{b}} f = \int_b^a f$	K2	CO3
4	7	Consider the following two subsets of vector space $V_2(R)$ $S_1 = \{(x_1, x_2)\} \mid x_1 + x_2 \geq 0$ $S_2 = \{(x_1, x_2)\} \mid x_1 + x_2 \leq 1$, then which of the following is satisfied (a) S_1 be a subspace of $V_2(R)$ but not S_2 (b) S_2 be a subspace of $V_2(R)$ but not S_1 (c) Both S_1 and S_2 be a subspace of $V_2(R)$ (d) Neither S_1 nor S_2 in a subspace of $V_2(R)$	K1	CO4
	8	Which of the following matrices is orthogonal? (a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$	K2	CO4
5	9	In the context of a square matrix A, an eigenvalue is a value λ that satisfies the equation _____________. (a) $Ax=x$ (b) $Ax=\lambda x$ (c) $A\lambda=x$ (d) $\lambda Ax=x$	K1	CO5
	10	A quadratic form Q(x) is positive definite if __________. (a) Q(x) ≥ 0 for all x (b) Q(x) < 0 for all x ≠ 0 (c) Q(x) = 0 for all x ≠ 0 (d) Q(x) > 0 for all x ≠ 0	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that the function $f(x) = x^2$ is continuous but not uniformly continuous on the interval $S=(0,\infty)$	K3	CO1
		(OR)		
	11.b.	Develop the short notes on maxima and minima of functions.		
2	12.a.	Prove that, if the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then $\{S_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L. That is, if $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} S_n = M$, then $L = M$.	K3	CO2
		(OR)		
	12.b.	State and prove Dini's theorem.		
3	13.a.	Let f be a bounded function on the closed bounded interval [a, b]. Then prove that $f \in \Re[a,b]$ if and only if f is continuous at almost every point in [a,b].	K5	CO3
		(OR)		
	13.b.	Develope the properties of Riemann – Stieltjes integral functions.		
4	14.a.	Prove that the intersection of any family of subspaces of a vector space V is a subspace of V.	K5	CO4
		(OR)		
	14.b.	Discuss the short notes on Inner product space.		
5	15.a.	Describe the classification of the quadratic form.	K4	CO5
		(OR)		
	15.b.	Prove that $h(\lambda_1), h(\lambda_2), \cdots, h(\lambda_n)$ are the characteristic root of $h(A)$, if $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the characteristic roots of A and let $h(\lambda)$ be a rational function such that $h(A)$ is defined. Then prove that $h(\lambda_1), h(\lambda_2), \cdots, h(\lambda_n)$ are the characteristic root of $h(A)$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove Taylor's theorem. Also write its significance.	K5	CO1
2	17	(i) State and prove Dirichlet's test. (ii) Prove that, if $\sum_{n=1}^{\infty} a_n$ is a convergent series, then $\lim_{n\to\infty} a_n = 0$.	K4	CO2
3	18	State and prove second mean value theorem.	K5	CO3
4	19	Prove that if $A = \{x_1, x_2, \cdots x_k\}$ and $B = \{y_1, y_2, \cdots y_{k+1}\}$ are linearly independent subsets of a vector space V, then there exists a $y_j \in B - A$ such that $A \cup \{y_j\}$ is linearly independent.	K4	CO4
5	20	State and prove Cayley-Hamilton theorem.	K5	CO5

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