22MAP105N/ 22MAP105

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024
(First Semester)

Branch – **MATHEMATICS**

MATHEMATICAL STATISTICS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Question No.	Question	K Level	CO
1	The coefficient of variation in the binomial distribution is (a) $\sqrt{npq}$ (b) np (c) $\sqrt{\frac{q}{np}}$ (d) $\sqrt{\frac{np}{q}}$	K1	CO1
2	Given random variable $x_1 = -1$ and $x_2 = +1$ with probabilities $p(x = -1) = p(x = +1) = 0.5$, then choose the characteristic function of this random variable. (a) $sint$ (b) $cost$ (c) $tan\,t$ (d) $sect$	K2	CO1
3	Choose the variance of Gamma distribution. (a) $\frac{p}{b^2}$ (b) $\frac{p}{b}$ (c) pb (d) pb^2	K1	CO2
4	In normal distribution μ_{2k+1} =______ for every k. (a) 0 (b) 1 (c) 2 (d) 3	K2	CO2
5	If X_1, X_2 ..... are independent random variable with the same distribution, whose standard deviation $\sigma \neq 0$ exists, then the sequence $\{F_n(z)\}$ of distribution functions of the random variable z_n, satisfies $\lim_{n\to\infty} F_n$ =______ (a) $\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz$ (b) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz$ (c) $\frac{1}{2\pi}\int_{-\infty}^{z} e^{-\frac{z^2}{2}} dz$ (d) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{z^2}{2}} dz$	K1	CO3
6	The random variable $X_n (n = 1,2 ....)$ are independent and each of them has the Poisson distribution given by $P(X_n = r) = \frac{e^{-2}2^r}{2!}$, then find standard deviation of Y_{100}. (a) 200 (b) 20 (c) $10\sqrt{2}$ (d) $10\sqrt{3}$	K2	CO3
7	Given $\{X_t, t \in I\}$ and $\{Y_t, t \in I\}$ be two real stochastic process, then $\{Z_t = X_t + iY_t, t \in I\}$ is a ______ (a) complex random variable (b) stochastic process (c) complex stochastic process (d) complex function	K1	CO4
8	The covariance of the complex random variable Z_1, Z_2 the components of the vector (Z_1, Z_2) is expressed by (a) $E((Z_1 - E(Z_1))(\overline{Z_2 - E(Z_2)})$ (b) $E((Z_1 + E(Z_1))(\overline{Z_2 - E(Z_2)})$ (c) $E((Z_1 - E(Z_1))(\overline{Z_2 + E(Z_2)})$ (d) $E((\overline{Z_1 - E(Z_1)})(\overline{Z_2 - E(Z_2)})$	K2	CO4
9	A random variable which is a function of the observed random vector $(X_1, X_2 \dots\dots X_n)$ is called a (a) sample (b) simple (c) statistic (d) degrees of freedom	K1	CO5
10	The characteristic function of a random variable with normal distribution $N(m; \frac{\sigma}{\sqrt{n}})$ is (a) $e^{itm-\frac{\sigma^2 t^2}{2n}}$ (b) $e^{it-\frac{\sigma t}{2n}}$ (c) $e^{it-\frac{i\sigma t}{2n}}$ (d) $e^{it^2-\frac{\sigma t}{2n}}$	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Define characteristic function. Also, discuss some properties of this function.	K3	CO1
	(OR)		
11.b.	The characteristic function of the random variable X is given by $\varphi(t) = e^{-\frac{t^2}{2}}$, then find the density function of this random variable.		
12.a.	Define one-point distribution and find the distribution function, characteristic function and variance of this distribution.	K3	CO2
	(OR)		
12.b.	(i) Explain Beta distribution (ii) Given X follows beta distribution with $p = q = 2$, Then find the probability that X is not greater than 0.2		
13.a.	State and prove Bernoulli law of large numbers.	K4	CO3
	(OR)		
13.b.	The random variable X_r where $r = 1,2$ ... have distinct values $k = 0,1,2 \ldots .9$ with the probabilities $p(x_r = k) = 0.1$ for every k. Consider the random variable $Y_{100} = \frac{X_1+X_2+\cdots+X_{100}}{100}$. Then analyze the probability that Y_{100} will exceed 5? $(p(0 < z < 1.74) = 0.4591)$		
14.a.	Explain Furry – Yule process.	K4	CO4
	(OR)		
14.b.	The function $R(\tau)$ is the correlation function of a process $\{Z_t, -\infty < t < +\infty\}$ stationary in the wide sense continuous and satisfying $m = 0, \sigma = 1$ if and only if there exist distribution function $(F(\lambda)$ such that $R(\tau) = \int_{-\infty}^{\infty} e^{i\lambda\tau}\, dF(\lambda)$.		
15.a.	Given sequence $\{F_n(t)\}$ of distribution functions of student's t with n degrees of freedom, then prove that $\lim_{n\to\infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2/2} dt.$	K4	CO5
	(OR)		
15.b.	Determine the mean of χ^2distribution.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	State and prove Chebyshev inequality.	K4	CO1
17	Define Poisson distribution. Also, determine characteristic function and variance of this distribution.	K4	CO2
18	State and prove De moivre-Laplace theorem.	K4	CO3
19	Prove that a stochastic process $\{X_t, 0 \le t < \infty\}$ where X_t is the number of signals in the interval [0, t) satisfying Poisson process and the equation $p(X_0 = 0) = 1$ is a homogeneous Poisson process.	K4	CO4
20	Define Fisher's Z distribution and derive the probability density function of this distribution.	K4	CO5

Z-Z-Z END