**22MAP422**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**MSc DEGREE EXAMINATION MAY 2024**
(Fourth Semester)

Branch – **MATHEMATICS**

**MATHEMATICAL METHODS**

Time: Three Hours  Maximum: 50 Marks

**SECTION-A (5 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (5 x 1 = 5)

1. A complex-valued function K(s,t) is called symmetric if ___________.
   (i) K(s,t)=K*(t,s) (ii) K(s,t)=K(t,s) (iii ) K(s,t)=K*(s,t) (iv) K(s,t)=1

2. In Fredholm's first theorem, the solution of the homogeneous equation $g(s) = \lambda \int K(s,t)\,dt$ is ____________.
   (i) zero (ii) identically zero (iii) one (iv) constant

3. An integral equation is called ________ if either the range of integration is infinite or the kernel has singularities within the range of integration.
   (i) constant (ii) non-singular (iii) singular (iv) symmetric

4. A function f(x) is called ___________ if to a small change of x there corresponds a small change in the function f(x).
   (i) continuous (ii) discontinuous (iii) differentiable (iv) analytic

5. If a proper field is formed by a family of extremals of a certain variational problem, then it is called an _________ field.
   (i) constant (ii) functional (iii) internal (iv) extremal

**SECTION - B (15 Marks)**
Answer **ALL** Questions
**ALL** Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Solve the Fredholm integral equation of the second kind
$$g(s) = s + \lambda \int_0^1 (st^2 + s^2 t) g(t)dt.$$
[OR]

   b) Solve the homogeneous Fredhlom integral equation $g(s) = \lambda \int_0^1 e^s e^t g(t)dt$.

7. a) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 e^{s-t} g(t)dt$.
[OR]

   b) Prove that the mth iterated kernel $K_m(s,t)$ satisfies the following relation: $K_m(s,t) = \int K_r(s,x)K_{m-r}(x,t)dx$, where r is any positive integer less than m.

8. a) Reduce the initial value problem $y''(s) + \lambda y(s) = F(s), y(0) = 1, y'(0) = 0$ to a Volterra integral equation.
[OR]

   b) Solve the integral equation $s = \int_0^s \frac{g(t)dt}{(s-t)^{1/2}}$.

**Cont...**

9. a) Prove that if a differentiable function $f(x)$ achieves a maximum or a minimum at an interior point $x = x_0$ of the domain of definition of the function, then at this point $df = 0$.

[OR]

b) Find the extremals of the functional

$v[y(x), z(x)] = \int_0^{\pi/2} [y'^2 + z'^2 + 2yz]dx, y(0) = 0, y\left(\frac{\pi}{2}\right) = 1, z(0) = 0, z\left(\frac{\pi}{2}\right) = -1.$

10.a) Is the Jacobi condition fulfilled for the extremal of the functional $v = \int_0^a (y'^2 - y^2)dx$ that passes through the points A(0,0) and B(a,0)?

[OR]

b) Test for an extremum the functional

$v[y(x)] = \int^a (y'^2 - y^2)dx,\ a > 0; y(0) = 0,\ y(a) = 0.$

**SECTION -C (30 Marks)**

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s + t)g(t)dt$ and find the eigen values.

[OR]

b) Show that the integral equation $g(s) = f(s) + (1/\pi) \int_0^{2\pi} [\sin(s + t)]g(t)dt$ possesses no solution for $f(s) = s$, but that it possesses infinitely many solutions when $f(s) = 1$.

12. a) Solve the Fredhlom integral equation $g(s) = 1 + \lambda \int_0^1 (1 - 3st)g(t)dt$ and evaluate the resolvent kernel.

[OR]

b) Evaluate the resolvent for the integral equation $g(s) = f(s) + \lambda \int_0^1 (s + t)g(t)dt$.

13. a) Reduce the boundary value problem $y''(s) + \lambda P(s)y = Q(s), y(a) = 0, y(b) = 0$ to a Fredholm integral equation.

[OR]

b) Solve the integral equation (a) $f(s) = \int_a^s \frac{g(t)dt}{(s^2-t^2)^\alpha}, 0 < \alpha < 1; a < s < b$, and

(b) $f(s) = \int_s^b \frac{g(t)dt}{(t^2-s^2)^\alpha}, 0 < \alpha < 1; a < s < b$.

14. a) State and prove the fundamental lemma of the calculus of variations.

[OR]

b) Derive Euler equation of the functional $F(x, y, y')$.

15.a) Test for an extremum the functional

$$v[y(x)] = \int_0^a \left(\frac{y}{y'^2}\right) dx,\ a > 0, 0 < b < 1; y(0) = 1, y(a) = b.$$

[OR]

b) Find the equation of geodesics on a surface on which the element of length of the curve is of the form $ds^2 = [\Phi_1(x) + \Phi_2(y)](dx^2 + dy^2)$, that is, find the extremals of the functional $S = \int_{x_0} \sqrt{[\Phi_1(x) + \Phi_2(y)](1 + y'^2)}dx$.

Z-Z-Z END