23MAP206/ 22MAP206

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2024
(Second Semester)

Branch – **MATHEMATICS**

COMPLEX ANALYSIS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The cross ratio (z_1, z_2, z_3, z_4) is the image of z_1 under the linear transformation which carries z_2, z_3, z_4 into …… a) $1,\infty,0$ b) $1,0,\infty$ c) $1,0,1$ d) $0,1,0$	K1	CO1
	2	The line integral is defined as $\int_{\gamma} f(z)dz =$ ……. a) $\int_a^b f(z(t))z'(t)\,dt$ b) $\int_a^b f(z(t))\,dt$ c) $\int_a^b z'(t)\,dt$ d) $\int_0^1 f(z(t))z'(t)\,dt$	K1	CO1
2	3	Which of the following is called Laplace's equation ? a) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ b) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial y}=0$ c) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$ d) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$	K1	CO2
	4	Which of the following is the Poisson integral ? a) $\frac{1}{2\pi}\int_0^{2\pi} \text{Re}\frac{e^{i\theta}+z}{e^{i\theta}-z}d\theta$ b) $\int_0^{2\pi} \text{Re}\frac{e^{i\theta}+z}{e^{i\theta}-z}U(\theta)d\theta$ c) $\frac{1}{2\pi}\int_0^{2\pi} \text{Re}\frac{e^{i\theta}+z}{e^{i\theta}-z}U(\theta)d\theta$ d) $\frac{1}{2\pi}\int_0^{2\pi} \text{Re}\frac{e^{i\theta}+2z}{e^{i\theta}-z}U(\theta)d\theta$	K2	CO2
3	5	$\Gamma(z)\Gamma(1-z)=$ a) $\sin\pi z$ b) $\cos\pi z$ c) $\frac{\pi}{\sin\pi z}$ d) $\frac{\pi}{\cos\pi z}$	K2	CO3
	6	$z-\frac{z^2}{2}+\frac{z^3}{3}-\frac{z^4}{4}+\ldots =$ a) $\sin z$ b) $\cos z$ c) $\log(1+z)$ d) $\log z$	K1	CO3
4	7	Which of the following is the Schwarz- Christoffel formula? a) $F(w)=C\int_0^w \prod_{k=1}^n (w)^{-\beta_k}dw+C'$ b) $F(w)=C\int_0^k \prod_{k=1}^w (w-w_k)^{-\beta_k}dw+C'$ c) $F(w)=C\int_0^w \prod_{k=1}^n (w-w_k)^{-\beta_k}dw$ d) $F(w)=C\int_0^w \prod_{k=1}^n (w-w_k)^{-\beta_k}dw+C'$	K1	CO4
	8	An analytic arc determined by $\varphi(t)$ is regular if a) $\varphi'(t)=0$ b) $\varphi'(t)\neq 0$ c) $\varphi(t)\neq 0$ d) $\varphi(t)=0$	K2	CO4
5	9	A nonconstant elliptic function has equally many poles as it has …. a) points b) regular c) poles d) zeros	K1	CO5
	10	The period of e^z is a) πi b) 2π c) π d) $2\pi i$	K2	CO5

Cont…

SECTION - B (35 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	If the line integral $\int_\gamma p\,dx + q\,dy$, defined in Ω, depends only on the end points of γ, then prove that there exists a function $U(x,y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$.	K3	CO1
		(OR)		
	11.b.	State and prove Liouville's theorem.		
2	12.a.	Evaluate $\int_0^\pi \frac{d\theta}{a+\cos\theta}, a > 1$.	K4	CO2
		(OR)		
	12.b.	If u_1 and u_2 are harmonic in a region Ω, then prove that $\int_\gamma u_1 {}^* du_2 - u_2 {}^* du_1 = 0$ for every cycle γ which is homologous to zero in Ω.		
3	13.a.	State and prove Hurwitz theorem.	K4	CO3
		(OR)		
	13.b.	Prove that the infinite product $\Pi_1^\infty(1+a_n)$ with $1+a_n \neq 0$ converges simultaneously with the series $\sum_1^\infty \log(1+a_n)$ whose terms represent the values of the principal branch of the logarithm.		
4	14.a.	Let f be a topological mapping of a region Ω onto a region Ω'. If $\{z_n\}$ or $z(t)$ tends to the boundary of Ω, then prove that $\{f(z_n)\}$ or $\{f(z(t))\}$ tends to the boundary of Ω'.	K5	CO4
		(OR)		
	14.b.	Prove that a continuous function $u(z)$ which satisfies $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + re^{i\theta})\,d\theta$ is necessarily harmonic.		
5	15.a.	Prove that an elliptic without poles is a constant.	K6	CO5
		(OR)		
	15.b.	Derive Legendre's relation.		

SECTION -C (30 Marks)
Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove Cauchy's theorem for a rectangle.	K4	CO1
2	17	State and prove the residue theorem.	K4	CO2
3	18	State and prove Mittag-Leffler theorem.	K4	CO3
4	19	State and prove the Riemann mapping theorem.	K4	CO4
5	20	Prove that any two bases of the same module are connected by a unimodular transformation.	K5	CO5

Z-Z-Z END