**18MCU32**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**BSc DEGREE EXAMINATION MAY 2024**
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

**DISCRETE MATHEMATICS AND GRAPH THEORY**

Time: Three Hours　　　　Maximum: 50 Marks

**SECTION-A (5 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks　　　　(5 x 1 = 5)

1 A _____ is a compound statement which is true for every value of the individual statements.
(i) negation　　(ii) tautology
(iii) contradiction　　(iv) equivalence

2 A compound proposition that is neither a tautology nor a contradiction is called __________
(i) Inference　　(ii) Equivalence
(iii) Condition　　(iv) Contingency

3 A relation R on a set A is said to be _____ if and only if the relation R is reflexive, symmetric and transitive
(i) binary relation　　(ii) an equivalence relation
(iii) composite relation　　(iv) an equivalent relation

4 A ______ graph is a graph in which each vertex is connected to every other vertex.
(i) regular　　(ii) connected
(iii)multi　　(iv) complete

5 A tree with four vertices has ____edges.
(i) 2　　(ii) 4　　(iii) 3　　(iv) 5

**SECTION - B (15 Marks)**
Answer **ALL** Questions
**ALL** Questions Carry **EQUAL** Marks　　　　(5 x 3 = 15)

6 a Prove $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$.

OR

b Obtain disjunctive normal form of $P \wedge (P \rightarrow Q)$.

7 a Show that $\neg(P \wedge Q)$ follows from $\neg P \wedge \neg Q$.

OR

b Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$.

8 a Let X={1,2,3,4} and R={ $\langle x,y \rangle$ | x>y}. Draw the graph of R and also give its matrix.

OR

b Let X={1,2,…,7} and R={ $\langle x,y \rangle$ | x-y is divisible by 3}. Show that R is an equivalence relation.

9 a Prove that the number of vertices of odd degree in a graph is always even.

OR

b Prove that a graph G is disconnected if and only if the vertex set V can be partitioned into two nonempty, disjoint subsets $V_1$ and $V_2$ such that there exists no edge in G whose end vertex is in subset $V_1$ and the other in subset $V_2$.

**Cont...**

10 a Prove that in a complete graph with n vertices there are (n-1)/2 edge-disjoint Hamiltonian circuits, if n is an odd number $\geq 3$.

OR

b Prove that a tree with n vertices has n-1 edges.

## SECTION -C (30 Marks)

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Obtain the principal disjunctive normal form of $P \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P)$.

OR

b Obtain the principal conjunctive normal form of $(\urcorner P \to R) \wedge (Q \leftrightarrow P)$.

12 a Show that the following are inconsistent.

1) If Jack misses many classes through illness, then he fails high school.
2) If Jack fails high school, then he is uneducated.
3) If Jack reads a lot of books, then he is not uneducated.
4) If Jack misses many classes through illness and reads a lot of books.

OR

b Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

13 a Let R and S be two relations on a set of positive integers I: R={ $\langle x, 2x \rangle$ | x ∈ I} S={ $\langle x, 7x \rangle$ | x ∈ I}. Find R ∘ S; R ∘ R; R ∘ R ∘ R and R ∘ S ∘ R.

OR

b Let f(x) = x+2, g(x) = x-2, and h(x) = 3x for x ∈ R, where R is the set of real numbers. Find g ∘ f; f ∘ g; f ∘ f; g ∘ g; f ∘ h; h ∘ g; h ∘ f and f ∘ h ∘ g.

14 a Explain Konigsberg Bridge problem.

OR

b Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

15 a Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree.

OR

b Prove that every tree has either one or two centers.

Z-Z-Z END