PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ADVANCED MATHEMATICAL STATISTICS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 The function $F(x,y)$ defined by $F(x,y) = P(X < x, Y < y)$ is called the ____________ of the random variable (X, Y)
(i) distribution function (ii) periodic function
(iii) convolution function (iv) normal function

2 What is the standard deviation of the uniform distribution?
(i) s (ii) a
(iii) σ (iv) λ

3 The sequence $\{X_n\}$ of random variables is called stochastically convergent to zero if for every $\varepsilon > 0$ the relation _____________ is satisfied.
(i) $\lim_{n\to\infty} P(|X_n| > \varepsilon) = 1$ (ii) $\lim_{n\to\infty} P(|X_n| > \varepsilon) = 0$
(iii) $\lim_{n\to\infty} P(|X_n| > 0) = 0$ (iv) $\lim_{n\to\infty} P(|X_n| > 1) = 0$

4 If _______ holds we say that the sequence $\{Z_n\}$ is convergent to zero alomost everywhere.
(i) $P\left(\lim_{n\to\infty} Z_n = 0\right) = 1$ (ii) $P\left(\lim_{n\to\infty} Z_n = 1\right) = 1$
(iii) $P\left(\lim_{n\to\infty} Z_n = \infty\right) = 1$ (iv) $P\left(\lim_{n\to\infty} Z_n = 0\right) = \infty$

5 A family of random variables X depending on the parameter t, where t belongs to a set I of real numbers is called ____________
(i) usual process (ii) random process
(iii) arbitrary process (iv) stochastic process

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Joint distribution of x and y is given by $f(x,y) = 4xy\,\bar{e}(x^2 + y^2), xzo,\ yzo$ Test whether x and y are independent.

OR

b Derive $\int_{\infty}^{\infty} f_1(x)\, f\left(\frac{y}{x}\right) dx = fz(y)$.

7 a Derive the mean and variance of rectangular distribution.

OR

b Narrate gamma distribution.

8 a Prove: Let $F_n(x)(n = 1,2,..)$ be the distribution function of the random variable X_n. The sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & for\ x \le 0 \\ 1 & for\ x > 0 \end{cases}$$

OR

b Prove: The sequence of random variables $\{X_n\}$ given $P\left(Y_n = \frac{r}{n}\right) = \binom{n}{r} p^r (1-p)^{n-r}$ and $X_n = Y_n - p$ is stochastically convergent to 0, that is for any $\varepsilon > 0$, we have $\lim_{n\to\infty} P(|X_n| > \varepsilon) = 0$

Cont

9 a Prove: Suppose that the random variable $X_1, X_2, \ldots$ are independent and have the same distribution with standard deviation $\sigma \neq 0$. Let the random variable U_n be defined by the formula $U_n = \frac{X_1+X_2+\cdots+X_n}{n}$. Furthermore, let $F_n(v)$ be the distribution function of the random variable V_n defined as $V_n = \frac{U_n - E(U_n)}{\sqrt{D^2(U_n)}}$. Then the sequence $\{F_n(v)\}$ satisfies the realtion $\lim_{n\to\infty} F_n(v) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{v} e^{-v^2/2} dv$

OR

b State and prove Lindeberg-Feller theorem.

10 a Summarize (i) Markov process (ii) Homogeneous.

OR

b Explain: A process with independent increments.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a If x and y are independent continuous random variables, then find the probability density functions of $Z = x - y$.

OR

b The distribution of the random variable (x,y) is given by the formulas $P(x = 1,\ y = 1) = P(x = 1,\ y = 2) = P(x = 2,\ y = 2) = 1/3$.

(i) Find the distribution functions $F(x, y),\ F_1(x)\ and\ F_2(y)$.

(ii) Check whether the points (1, ½), (1,3), (2, ½) and (2,3) are discontinuous point on f(x,y).

12 a Explain: Beta distribution with derivation.

OR

b Explain: Cauchy distribution with density function and characteristic function.

13 a Prove: If the sequence $\{F_n(x)\} (n = 1,2,..)$ of distribution functions is convergent to the distribution function $F(x)$, then the corresponding sequence of characteristic functions $\{\phi_n(t)\}$ converges at every point t, $(-\infty < t < \infty)$ to the function $\phi(t)$ which is the characteristic function of the limit distribution function $F(x)$, and the convergence to $\phi(t)$ is uniform with respect to t in every finite interval on the t-axis

OR

b Prove: If the sequence of characteristic functions $\{\phi_n(t)\}$ converges at every point $t(-\infty < t < \infty)$ to a function $\phi(t)$ continuous in some interval $|t| < \tau$, then the sequence $\{F_n(x)\}$ of corresponding distribution functions converges to the distribution function $F(x)$ which corresponds to the characteristic function $\phi(t)$

14 a State and prove Lapunov theorem.

OR

b Prove: If a random variable X of the discrete type takes on with positive probability only integer values and satisfies condition (w), then its characteristic function satisfies the relations $\phi(2\pi) = 1, |\phi(t)| < 1$ if $0 < |t| < 2\pi$

15 a Prove: A stochastic process $\{X_t, 0 \leq t < \infty\}$ where X_t is the number of singals in the interval $[0, t)$, satisfying conditions I to III and the equality $P(X_0 = 0) = 1$, is a homogeneous Poisson process.

OR

Prove: The solution $V_m(t)$ of the system $V_0'(t) = -\lambda_i V_0(t)$,
$V'_m(t) = -\lambda_{i+m} V_m(t) + \lambda_{i+m-1} V_{m-1}(t)$ $(m = 1,2,..)$ with the intial conditions $= \begin{cases} 1 & for\ m = 0 \\ 0 & for\ m \neq 0 \end{cases}$ satisfy the relation $\sum_{m=0}^{\infty} V_m(t) = 1$ if and only if
$= \infty$

Z-Z-Z END