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22STP102/18STP02

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(First Semester)

Branch – **STATISTICS**

ADVANCED PROBABILITY THEORY/ PROBABILITY THEORY

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. Probability is
   (i) a measure of event
   (ii) a measure of the chances of occurrence of events
   (iii) a measure of experiment
   (iv) a measure of trial

2. Characteristic function of X is
   (i) E (exp(iux)) (ii) E (exp(ixy)) (iii) E (exp(xy)) (iv) E (explux))

3. If X and Y are said to be uncorrelated
   (i) Cov(x,y) >0 (ii) Cov(x,y) ≠0 (iii) Cov(x,y) =0 (iv) Cov(x,y) <0

4. If $X_{m+n} - X_n \xrightarrow{P} 0$
   (i) as m,n $\to -\infty$ (ii) as m,n $\to \infty 0$ (iii) as m,n $\to 1$ (iv) as m,n $\to \infty$

5. If $\{X_k\}$ is a sequence of independent Bernoulli random variable, then
   $Sn - np / \sqrt{npq}$ is converges in Law to ---------------.
   (i) Normal variate (ii) Standard normal variate
   (iii) Bernoulli variate (iv) Binomial variate

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Give an example for countable probability space.

   OR

   b) List out the linear properties of Expectation.

7. a) Drive the characteristic function for the r.v variable distributed as standard normal.

   OR

   b) Show that characteristic function is e^{-14} if $f(x)$ be the density function of the distribution .

8. a) If X and Y are true arbitrary, show that E $(x\,y)$ = E(x).E(y).

   OR

   b) If X_i's are independent standard normal variates, show that $\sum X_i^2$. is distributed as chi-square with n degree of freedom.

9. a) If E $|Xn|^r \to 0$ show that $X_n \xrightarrow{P} 0$

   OR

   b) If $X_n \xrightarrow{P} x$ and $Y_n \xrightarrow{P} Y$ show that $X_n + Y_n \xrightarrow{P} x + y$

10. a) If the series $\Sigma\sigma_k^2 < \infty$, show that $\Sigma(X_k\text{-}EX_k)$ converges in probability.

    OR

    b) Explain Bernoulli weak law of large numbers.

Cont...

SECTION -C (30 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) State and prove the Holder's inequality.

OR

b) Outline the concept of finite probability space.

12. a) State and prove the Bochner's theorem.

OR

b) State and prove Levy's continuity theorem.

13. a) Explain the independence classes.

OR

b) State and prove Borel a.s criterion.

14. a) Prove that Fn(X) $\rightarrow F(x), x \in C(F) if\ Xn \xrightarrow{P}$ X.

OR

b) State and prove Helly Bray lemma.

15. a) If $\Sigma\sigma_n^2 < \infty$, then $\Sigma_n (X_n - EX_n)$ converges a.s. Prove that converge is also true, if X_n's is a.s bounded.

OR

b) State and prove Lindeberg –Levy Theorem.

Z-Z-Z END