20MAP21

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(Fourth Semester)

Branch – **MATHEMATICS**

MATHEMATICAL METHODS

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 A complex-valued function $K(s,t)$ is called ________ if $K(s,t) = K^*(s,t)$, where * denotes complex conjugate.
(i) separable (ii) symmetric
(iii) discontinuous (iv) skew-symmetric

2 If $g(s)=\lambda\int_0^{\pi}(\sin s\ \sin 2t)g(t)dt$ has ---------- eigen values.
(i) No (ii) two
(iii) one (iv) none of these

3 Initial value problem is reduced to ----------------- integral equation with given initial conditions.
(i) Fredholm Type (ii) Volterra Type
(iii) Singular (iv) both

4 The line of quickest descent is a ---------.
(i) cardiod (ii) cycloid
(iii) catenoid (iv) straight line

5 The slope of the tangent line $p(x,y)$ to the curve of the family $y = y(x, C)$ that passes through the point (x, y) is called the ______at the point (x, y).
(i) central field (ii) field of extremals
(iii) slope of the field (iv) family of straight lines

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a) Solve the Fredholm integral equation $g(s) = s + \lambda\int_0^t(st^2 + s^2t)\, g(t)dt$.
(OR)
b) State and Prove Fredholm Alternative Theorem.

7 a) Find the Neumann series for the solution of the integral equation
$$g(s) = (1 + s) + \lambda\int_0^s(s - t)g(t)dt.$$
(OR)
b) Solve the integral equation $g(s)= f(s) + \lambda\int_0^1(e^{s-t})g(t)dt$ and evaluate the resolvent kernal.

8 a) Reduce the Initial value problem $y''(s)+ \lambda y(s)=F(s)$ with the initial conditions $y(0)=1, y'(0)=0$ to a volterra integral equation.
(OR)
b) Solve the Abel's Integral equation $f(s) = \int_0^s \frac{g(t)}{(s-t)^{\alpha}}dt, 0<\alpha<1$.

Cont...

9 a) State and prove the fundamental Lemma of calculus of variations .

(OR)

b) On what curves can the functional $V[y(x)] = \int_0^1 (y'^2 + 12xy)\, dx; y(0) = 0,$ $y(1) = 1$ be extremized?

10 a) Is the Jacobi condition fulfilled for the extremal of the functional $V[y(x)] = \int_0^a (y'^2 - y^2)\, dx$ that passes through the points A(0,0) and B(a, 0)?

(OR)

b) Test for an extremum the functional $V[y(x)] = \int_0^a y'^3 dx;\ y(0) = 0\ y(a) = b, a > 0, b > 0.$

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a) Find the eigenvalues and eigen functions of the homogeneous integral equation g(s) = $\lambda \int_1^2 [st + (1/st)] g(t) dt.$

(OR)

b) Solve the integral equation g(s) = f(s) + $\frac{1}{\pi}\int_0^{2\pi} \sin(s+t)\, g(t)\, dt$ possess no solution for f(s) = s, but that it possesses infinitely many solutions when f(s) = 1.

12 a) Solve the integral equation g(s)=(e^s –s) - $\int_0^1 s(\ e^{st-1}) g(t) dt$ using approximation method.

(OR)

b) Find the resolvent kernel for the integral equation g(s) = f(s) + λ $\int_{-1}^1 (st + s^2t^2) g(t)\, dt.$ Also find the solution when $D(\lambda) = 0.$

13 a) State and solve the Transverse oscillation problem of a homogeneous elastic bar.

(OR)

b) Solve the Abel's integral equation f(s) = $\int_s^b \frac{g(t)}{[h(t)-h(s)]^\alpha}$, 0<α<1, a<s<b.

14 a) Derive Euler's equation.

(OR)

b) State and Prove Brachistrone problem.

15 a) Discuss briefly about transforming the Euler equations to the canonical form.

(OR)

b) Derive Hamilton-Jacobi equation.

Z-Z-Z END