22MAP208

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(Second Semester)

Branch – **MATHEMATICS**

TOPOLOGY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 If X is any set , then the collection of all subsets of X is a topology on X and is called
(i) discrete topology (ii) trivial topology
(iii) indiscrete topology (iv) standard topology

2 Which one of the following is standard bounded metric ?
(i) $\bar{d}(x,y)=\min\{d(x,y),-1\}$ (ii) $\bar{d}(x,y)=\min\{d(x,y),1\}$
(iii) $\bar{d}(x,y)=\max\{d(x,y),1\}$ (iv) $\bar{d}(x,y)=\max\{d(x,y),0\}$

3 A space X is said to be connected if there
(i) exists a separation of X (ii) does not exist a separation of X
(iii) exists a continuous map on X (iv) does not exist a continuous map on X

4 A subset A of a space X is said to be dense in X if
(i) $\bar{A}=X$ (ii) $\bar{A}=\bar{X}$
(iii) $A=X$ (iv) $\bar{A}\subset X$

5 A space X is said to be …….. if for each pair A, B of disjoint closed sets of X, there exist disjoint open sets containing A and B respectively.
(i) regular (ii) normal
(iii) Hausdorff (iv) connected

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Let A be a subset of the topological space X and let A' be the set of all limit points of A . Then prove that $\bar{A}=A\cup A'$.

OR

b If $\mathcal{B}$ is a basis for the topology of X and $\mathbb{C}$ is a basis for the topology of Y, then prove that the collection $D=\{B\times C|\ B\in\mathcal{B}\ and\ C\in\mathbb{C}\}$ is a basis for the topology of $X\times Y$.

7 a State and prove the pasting lemma.

OR

b State and prove the sequence lemma.

8 a Prove that the image of a connected space under a continuous map is connected.

OR

b State and prove uniform continuity theorem.

9 a Prove that compactness implies limit point compactness.

OR

b Suppose that X has a countable basis. Then prove that every open covering of X contains a countable subcollection covering X .

Cont...

10 a Prove that every metrizable space is normal.

OR

b Prove that a product of completely regular space is completely regular.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Let A be a subset of a topological space X. Prove that

i) $x \in \bar{A}$ if and only if every open set U containing x intersects A.

ii) Supposing the topology of X is given by a basis then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.

OR

b i) If $\mathcal{B}$ is a basis for the topology τ of a set X, then prove that τ equals the collection of all unions of elements of $\mathcal{B}$.

ii) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ', respectively, on X. Then prove that the τ' is finer than τ iff for each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $X \in B' \subset B$.

12 a Prove that the topologies on R^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on R^n.

OR

b Let X and Y be topological spaces and $f: X \to Y$. Then prove that the following are equivalent.

(i). f is continuous.

(ii). For every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$.

(iii). For every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

(iv). For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

13 a Prove that if L is a linear continuum in the order topology, then L is connected and so are intervals and rays in L.

OR

b Prove that the product of finitely many compact spaces is compact.

14 a i) Prove that a subspace of a Hausdorff space is Hausdorff and a product Hausdorff spaces is Hausdorff.

ii) Prove that a subspace of a regular space is regular and a product of regular space is regular.

OR

b In a metrizable space X, prove that the following are equivalent.

(i). X is compact.

(ii). X is limit point compact.

(iii). X is sequentially compact.

15 a State and prove the Urysohn lemma.

OR

b State and prove the Tychonoff theorem.

Z-Z-Z END