22MAP102

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(First Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. If f is differentiable in $(a, b), f'(x) \leq 0$ for all in $x \in (a, b)$ then f is
   (i) monotonically increasing (ii) constant
   (iii) monotonically decreasing (iv) continuous at x

2. $\int_{-a}^{b} f\, d\alpha,\ \int_{a}^{-b} f\, d\alpha$ are lower and upper Riemann integrals on $[a, b]$ then
   (i) $\int_{-a}^{b} f\, d\alpha \geq \int_{a}^{-b} f\, d\alpha$ (ii) $\int_{-a}^{b} f\, d\alpha = \int_{a}^{-b} f\, d\alpha$
   (iii) $\int_{-a}^{b} f\, d\alpha \leq \int_{a}^{-b} f\, d\alpha$ (iv) None of the above

3. What is value of $\lim_{n\to\infty} f_n\left(\frac{1}{n}\right)$, where $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}$
   (i) 0 (ii) 0.5 (iii) -0.5 (iv) 1

4. $E(\pi i) =$
   (i) i (ii) $-i$ (iii) 1 (iv) -1

5. If $f'(x) = 0$ for all $x \in E$ then f is
   (i) constant (ii) polynomial
   (iii) quadratic (iv) linear

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. (a) Show that f is a differentiable at all points x but f' is not a continuous function for $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

   (OR)

   (b) State Taylor's Theorem.

7. (a) If $f, g \in R(\alpha)$ on $[a, b]$ then $fg \in R(\alpha)$ Justify your answer.

   (OR)

   (b) State Fundamental theorem of calculus and apply it to prove Integration by parts.

8. (a) State Weierstrass test foe sequence of functions defined on a set E.

   (OR)

   (b) Is there is a sequence of continuous functions converge to a continuous function but not uniformly? Jusitify your answer.

9. (a) Define Trigonometric polynomial and find its periodic.

   (OR)

   (b) Prove that $\log \Gamma$ is convex on $(0, \infty)$.

10. (a) If P is a projection in X then show that every $x \in X$ has a unique representation of the form
$X = x_1 + x_2$ where $x_1 \in Rank(P), x_2 \in$ Nullity (P).

(OR)

(b) Define differentiable function in an open set E of R^n

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. (a) State Generalized mean value theorem and construct mean value theorem from it.

(OR)

(b) Derive L' Hospital Rule.

12. (a) If γ' is continuous on $[a, b]$ then γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

(OR)

(b) Construct the necessary and sufficient condition for a function is Riemann Integrable On $[a, b]$.

13. (a) Is there is a sequence $\{n_k\}$ such that $\{f_n(x)\}$ converges where $f_n(x) = \sin nx \quad (0 \le x \le 2\pi, n = 1,2,3 \ldots)$? Justify your answer.

(OR)

(b) State and Prove Stone- Weierstrass theorem.

14. (a) If f is a positive function on $(0, \infty)$ such that $f(x+1) = x.f(x),\ f(1) = 1, \log f$ is a Constant then prove that $f(x) = \Gamma(x)$

(OR)

(b) State and prove Taylor theorem.

15 (a) Show that $\dim R^n = n$

(OR)

(b) Construct necessary and sufficient condition for a linear operator on a finite dimensional vector space X is one-to-one.

Z-Z-Z END