11BMC24

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION JUNE 2014
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

COMPLEX ANALYSIS

Time : Three Hours | Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define analytic.
2 Show that the function $u=x^3-3xy^2$ is harmonic and find the corresponding analytic function.
3 Define critical points and ordinary points.
4 State the necessary conditions for W=f(z) to represent a conformal mapping.
5 Using the definition of an integral as the limit of a sum, evaluate $\int_2 |dz|$.
6 State Cauchy's fundamental theorem.
7 Show that when $0<|z|<4$, $\frac{1}{4z-z^2}=\sum_{n=0}^{\infty}\frac{z^{n-1}}{4^{n+1}}$.
8 State fundamental theorem of Algebra.
9 State Cauchy's Residue theorem.
10 State Jordan's inequality.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that the function $|z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

OR

b Prove that the real and imaginary parts of an analytic function satisfy Laplace's equation.

12 a What is the region of the w-plane into which the rectangular region in the z-plane bounded by the lines x=0,y=0,x=1 and y=2 is mapped under the transformation w=z+(2-i)?

OR

b Consider the transformation W=2z and determine the region D' of the w-plane into which the triangular region D enclosed by the lines x=0,y=0,x+y=1, in the z-plane is mapped under this transformation.

13 a Prove that the value of the integral of $\frac{1}{z}$ along a semi-circular arc $|z|=1$ from -1 to +1 is $-\pi i$ or πi according as the arc lies above or below the real axis.

OR

b State and prove Morera's theorem.

14 a State and prove Liouville's theorem.

OR

b Prove that a function which has no singularity in the finite part of the plane or at infinity in constant.

Cont...

15 a Evaluate the residues of $\frac{z^3}{(z-1)(z-2)(z-3)}$ at 1,2,3 and infinity and show that their sum is zero.

OR

b Show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{(a^2-b^2)}$, a>b>0.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 If $u+v = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$, find the analytic function f(z)=u+iv.

17 Let f(z) be an analytic function of z in a region D of the z-plane and let $f'(z) \neq 0$ in D. Then prove that the mapping w=f(z) is conformal at all points of D.

18 State and prove Cauchy's Integral formula.

19 State and prove Laurent's theorem.

20 Prove that

i) $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}$ ii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3} = \frac{3\pi}{8}$.

Z-Z-Z END