PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017
(First Semester)

Branch – **MATHEMATICS**

CLASSICAL ALGEBRA & TRIGONOMETRY

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define a sequence, by means of an example explain the limit of htat sequence.

2 What can be said about
i) $\{a_n - b_n\}$ when $\{a_n\}$ & $\{b_n\}$ are divergent
ii) $\{a_n b_n\}$ when $\{a_n\}$ converges to 'a' and $\{b_n\}$ conveyers to 'b'.

3 If $\sum u_n$ and $\sum v_n$ are 2 series of positive terms and $\frac{u_{n+1}}{u_n} < \frac{v_{n+1}}{v_1}$ and $\frac{u_1}{v_1}$ is a constant when can $\sum u_n$ be convergent?

4 State Weissstrass M test for uniform convergence.

5 What can you say about the roots of f(x) = 0 when f(a) and f(b) have unlike signs?

6 If $\alpha_1, \alpha_2, \ldots\ldots \alpha_n$ are roots of $x^n + p_1x^{n-1} + p_2x^{n-2} + \ldots\ldots + p_n = 0$ find $\sum \alpha_1$ and $\sum \alpha_2$.

7 State De'moivre's theorem & hence write the expansion of Cos n θ.

8 Write the expansion of sin θ & tan θ in terms of '0'.

9 The coefficient of 'i' in the principal value of log (x + iy) lies between _____ and _____.

10 Log (-x) = _____.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a If $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots + \frac{1}{2n}$, show that the sequence $\{a_n\}$ tends to a limit.

OR

b Prove $\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \frac{4}{7.9} + \ldots$ is divergent.

12 a Prove $1 + \frac{1}{2}.\frac{1}{3} + \frac{1.3}{2.4}.\frac{1}{5} + \frac{1.3.5}{2.4.6}.\frac{11}{7} + \ldots\ldots$ converges.

OR

b Test for convergency and divergency of $1 + \frac{2x}{2!} + \frac{3^2x^2}{3} + \frac{4^3x^3}{4!} + \frac{5^4x^6}{5!} + \ldots\ldots$

13 a Solve $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in arithmetic progression.

OR

b If α, β, χ, δ are roots of $x^4 + px^3 + qx^2 + rx + s = 0$ find (i) $\sum \alpha^2$ (ii) $\sum \alpha^2 \beta^2$.

Cont ...

14 a Prove $\cos h\, 2e = \dfrac{1+\tanh^2 x}{1-\tanh^2 x}$.

OR

b If θ is small prove $\theta \cot 0 = 1 - \theta^2/3 - \theta^6/45$ approximately.

15 a Prove $i^{i} = e^{-(4n+1)\pi/2}$ where n is an integer.

OR

b Sum to n terms the series $\sin^2\alpha + \sin^2 2\alpha + \ldots\ldots$

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Examine the Convergence of $\sum \dfrac{(n+1)(n+2)\ldots(n+n)}{n^n}$.

17 Show that the series $\dfrac{1}{1^k} + \dfrac{1}{2^k} + \dfrac{1}{3^k}$ is convergent when k is greater than unity and divergent when $k \leq 1$.

18 Find the sum of the cubes of the root of $x^5 = x^2 + x + 1$.

19 Prove $\cos 8\theta = 1 - 32\sin^2\theta + 160\sin^4\theta - 25\sin^6\theta + 128\sin^8\theta$.

20 Show $\log \tan\left(\pi/4 + ix/2\right) = i\tan^{-1}(\sinh x)$.

Z-Z-Z END