(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017

(First Semester)

Branch – **MATHEMATICS**

CALCULUS

Time : Three Hours — Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Give the rule for determining the maxima and minima values of f(x) when f(x) and f$'$(x) are continuous.

2 Find the maximum value of $\frac{\log x}{x}$ for positive values of x.

3 Define evolute and involute.

4 Give the coordinates of the centre of curvature.

5 Prove that $\int_0^{\pi/2} \sin^n x \; dx = \int_0^{\pi/2} \cos^n x \; dx$.

6 If f(x) is an odd function of x, then prove that $\int_{-a}^{a} f(x)dx = 0$.

7 Evaluate $\iint xy \, dx \, dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.

8 Define double integral.

9 Prove that $\Gamma(n+1) = n!$ when n is a positive integer.

10 Evaluate $\int_0^1 x^7 (1-x)^8 dx$.

SECTION - B (25 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find $\frac{du}{dt}$ where $u = x^2 + y^2 + z^2$, $x = e^t$, $y = e^t \sin t$, $z = e^t \cos t$.

OR

b Find $\frac{du}{dx}$ where $u = x^2 + y^2$ where $y = \frac{1-x}{x}$.

12 a Find the envelope of the family of circles $(x-a)^2 + y^2 = 2a$, where a is the parameter.

OR

b What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1)?

13 a Prove that $\int_0^{\pi} \theta \sin^3 \theta d\theta = \frac{2\pi}{3}$.

OR

b Find the reduction formula for $\int \cos^n x \; dx$ (n being a positive integer).

Cont ...

14 a Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the region for which x, y are each ≥ 0 and $x+y \leq 1$.

OR

b Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy\,dx\,dy$ and evaluate it.

15 a Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

OR

b Express $\int_0^1 x^m(1-x^n)^p dx$ interms of Gamma functions and evolute the integral $\int_0^1 x^5(1-x^3)^{10} dx$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find the maximum or minimum values of $2(x^2 - y^2) - x^4 + y^4$.

17 Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid.

18 Find the reduction formula for $I_{m,n} = \int x^m (\log x)^n dx$ (where m and n are positive integers). Hence or otherwise evolute $\int x^4 (\log x)^3 dx$.

19 Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

20 Prove that $\iiint \frac{dxdydz}{(1-x^2-y^2-z^2)^{1/2}} = \frac{\pi^2}{8}$, the integration extended to all positive values of the variables for which the expression is real.

Z-Z-Z END