PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017
(Second Semester)

Branch – **MATHEMATICS**

ANALYTICAL GEOMETRY OF 3D & VECTOR CALCULUS

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1 Find the equation of the sphere whose diameter is the join of (2,-3,1) and (1,-2,-1).

2 Find the equation of a sphere which passes through the point (1,-2,3) and the circle $z=0$, $x^2+y^2+z^2-9=0$.

3 Find the equation of the cone whose vertex is the origin and whose base is the circle $x^2+y^2+z^2=25$, $x+2y+2z=9$.

4 Prove that the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$, where $3l^2+3m^2-5n^2=0$ is a generator of the cone $2x^2+3y^2-5z^2=0$

5 Write the equation of the tangent plane of the cone.

6 Write the equation of the tangent plane to the conicoid $ax^2+by^2+cz^2=1$.

7 Find the magnitude and the direction of the greatest change of $u=xyz^2$ at (1,0,3).

8 If A and B are irrotational, show that A x B is solenoidal.

9 Given the vector field $\bar{F}=xz\bar{i}+yz\bar{j}+z^2\bar{k}$ evaluate $\int_c \bar{F}.d\bar{r}$ from the point (0,0,0) to (1,1,1) where c is the curve $x=t$, $y=t^2$, $z=t^3$.

10 State the Gauss Divergence Theorem.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find the equation to the sphere through the four points (0,0,0), (a,0,0), (0,b,0) and (0,0,c).

OR

b Show that the sphere $x^2+y^2+z^2-2x+6y+14z+3=0$ divides the line joining the points (2,1,-4) and (5,5,5) internally and externally in the ratio 1:2.

12 a Prove that the cones $3x^2+4y^2+5z^2+2yz+4zx+6xy=0$ and $19x^2+11y^2+3z^2+6yz-10zx-26xy=0$ are reciprocal.

OR

b Find the equations to the lines in which the plane $2x-3y+z=0$ cuts the cone $3x^2-5y^2-7z^2+36yz-20zx-2xy=0$. Hence find the angle between them.

13 a Find the equation to the cylinder whose generators are parallel to $x=\frac{y}{-2}=\frac{z}{3}$ and whose base curve is $x^2+2y^2=1$; $z=3$.

OR

b Find the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y=1$ having its generators parallel to $x=y=z$.

14 a Find $\nabla\circ\bar{F}$ and ∇ x F of the vector point function $\bar{F}=xz^3\bar{i}-2x^2yz\bar{j}+2yz^4\bar{k}$ at the point (1,-1,1).

OR

b Find the value of the arbitrary constant 'a' so that curl of the vector $\bar{F}=\left(axy-z^3\right)\bar{i}+(a-2)x^2\bar{j}+(1-a)xz^2\bar{k}$ is zero.

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15 a Evaluate $\int_c \overline{F}.d\overline{r}$, where $\overline{F} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$ and c is the straight line from A(0,0,0) to B(2,1,3).

OR

b Use divergence theorem, evaluate $\iint \nabla r^2 \cdot \hat{n} ds$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Show that the circles, $x^2+ y^2+z^2 -2x+3y+4z-5=0$, $5y+6z+1=0$
$x^2+ y^2+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$ lie on the same sphere and find its equation.

17 (i) Define a Cone

(ii) Find the equation of the right circular cone which passes through the line $2x=3y=-5z$ and has $x=y=z$ as its axis.

18 Find the equation of the right circular cylinder whose axis is $x=2y=-z$ and radius 4. Find also the area of the section of the cylinder by the plane $x \circ y$.

19 (i) Find the unit vector normal to the surface $\phi = x^3 - xyz + z^3 - 1$ at the point (1,1,1).

(ii)Find the angle of intersection at the point (2,-1,2) of the surfaces $x^2+ y^2+z^2=9$ and $z=x^2+y^2-3$.

20 Evaluate $\iint_s \overline{F}.\ \hat{n} ds$ where $\overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k}$ and s is the surface of the cube bounded by x=0,x=1,y=0,y=1,z=0,z=1.

Z-Z-Z END