PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017
(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time : Three Hours — Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 When do you say that the sets A and B have the same cardinal number?

2 Prove that a finite point set has no limit point.

3 Define a compact set.

4 A and B are two subsets of a metric space X. When do you say that they are separated?

5 Define the diameter of a non-empty set E of a metric space X.

6 If $\sum a_n$ converges, prove that $\lim_{n \to \alpha} a_n = 0$.

7 Define : Uniformly continuous.

8 Define : Discontinuity of the first kind.

9 Suppose f and g are defined on [a, b] and are differentiable at a point $x \in [a, b]$. Write the formula for $\left(\frac{f}{g}\right)'(x)$.

10 State : Generalized mean value theorem.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that every infinite subset of a countable set A is countable.

OR

b Define closure of E. Prove that $E = \overline{E}$, if and only if E is closed.

12 a If E is an infinite subset of a compact set k, prove that E has a limit point in k.

OR

b Let E be a subset of the real line R^1. If $x, y \in E$, $x < z < y$ and E is connected, prove that $z \in E$.

13 a If $0 \leq x < 1$, prove that $\sum_{n=0}^{\infty} x_n = \frac{1}{1-x}$. If $x \geq 1$, prove that the series diverges.

OR

b Prove that e is irrational.

Cont ...

14 a F is a mapping of a metric space X into a metric space Y, If F is continuous on X, prove that $f^{-1}(v)$ is open in X, for every open set V in Y.

OR

b Let X and Y be two metric spaces. If f is a continuous mapping of X into Y and if E is a connected subset of X, prove that f (E) is connected.

15 a Let f be defined on [a, b]. If f is differentiable at a point $x \in [a, b]$, prove that f is continuous at "x". What do you say about the converse?

OR

b Let f be defined as $f(x) = \begin{cases} x.\sin\left(\frac{1}{x}\right), & (x \neq 0) \\ 0, & (x = 0) \end{cases}$ show that f '(x) does not exist when x = 0 and find f '(x), if $x \neq 0$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Let $\{E_\alpha\}$ be a (finite or infinite) collection of sets E_α.

Then $\left(\bigcup_\alpha E_\alpha\right)^c = \bigcap_\alpha \left(E^c_\alpha\right)$.

17 Suppose $K \subset Y \subset X$. Then K is compact relative to X if and only if K is compact relative to Y.

18 Let $\{S_n\}$ be monotonic. Prove that $\{S_n\}$ converges if and only of it is bounded.

19 Let E be a non compact set in R^1. Then prove the following

a) There exists a continuous function on E which is not bounded.

b) There exists a continuous and bounded function on E, which has no maximum.

20 State and prove the mean value theorem.

Z-Z-Z END