BSc DEGREE EXAMINATION DECEMBER 2017
(Third Semester)

Branch – **MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATIONS & FOURIER TRANSFORM

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1   Eliminate the arbitrary function from $z = f(x^2 + y^2)$.

2   Solve $\dfrac{\partial^2 z}{\partial x \partial y} = 0$.

3   Write the complete integral of $px + qy + f(p, q) = 0$.

4   Write the subsidary equation of $(y + z) p + (z + x) q = x + y$.

5   Define Fourier series.

6   State Drichelet's conditions.

7   Prove that Fourier transform is linear.

8   Define infinite complex Fourier transform.

9   Find Fourier sine and cosine transforms of $f''(x)$.

10   Define inversion formula for sine transform.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a   Solve $p^2 + q^2 = npq$.

    OR

    b   Solve $9(p^2 z + q^2) = 4$.

12 a   Find the general solution of $(y + z) p + (z + x)q = x + y$.

    OR

    b   Solve $p^2 + x^2 y^2 q^2 = x^2 z^2$.

13 a   Express $f(x) = x \ (-\pi < x < \pi)$ as a Fourier series with period 2π.

    OR

    b   Express $f(x) = c - x$ where $0 < x < c$ as a half range cosine series with period $2c$.

14 a   Show that the transform $e^{-x^2/2}$ is $e^{-s^2/2}$ by finding the fourier transform of $e^{-a^2 x^2}$, $a > 0$.

    OR

    b   Find Fourier cosine transform of $f(x) = \begin{cases} \cos x & \text{in } 0 < x < a \\ 0 & x \geq a \end{cases}$

15 a Find the finite Fourier cosine transform of $f(x) = e$ in $(0.)$

<div align="center">OR</div>

b Solve the diffusion equation $\dfrac{\partial u}{\partial t} = K\dfrac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$, $t > 0$, with the

conditions. $u(x, 0) = f(x)$ and $\dfrac{\partial u}{\partial x}$, u tend to zero as x tends to $+\infty$.

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SECTION - C (30 Marks)
Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

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16 Solve (i) $Z = px + qy + p^2q^2$ (ii) $p^2 + q^2 = x + y$.

17 Obtain a complete integral of $xp^2 - ypq + y^3q - y^2z = 0$.

18 Express $f(x) = \dfrac{1}{2}(\pi - x)$ as a Fourier series with period 2π, to be valid in the interval 0 to 2π.

19 State and prove Parseval's identity.

20 Solve $\dfrac{\partial u}{\partial t} = 2\dfrac{\partial^2 u}{\partial x^2}$, $0 < x < 4$, $t > 0$ given $u(0, t) = 0$, $u(4, t) = 0$;

$u(x, 0) = 3 \sin \pi x - 2 \sin 5\pi x$.

<div align="center">Z-Z-Z   END</div>