PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017
(Fifth Semester)

Branch – **MATHEMATICS**

ALGEBRA - I

Time : Three Hours — Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Let $\sigma : S \to T$ be a mapping where
S=set of real numbers
T=set of non-negative real numbers.
Define σ by $s\sigma = s^2$ for all $s \in S$. Is σ a one-to-one mapping? Justify your answer.

2 Is H is a subgroup of an abelian group G, show that H is a normal subgroup of G.

3 Define order of an element in a group G.

4 G=group of integers under addition and let $G = \overline{G}$. Define $\phi : G \to \overline{G}$ by $\phi(x) = 2x$. Show that ϕ is a homomorphism.

5 State Cayley's theorem.

6 Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 1 & 2 & 3 \end{pmatrix}$

7 Define integral domain and give one example.

8 If R is a ring and $\phi : R \to R$ is a homomorphism show that $\phi(0) = 0$

9 Define maximal ideal in a ring R.

10 Let R be an integral domain with unit element and suppose that for $a, b \in R$, both a divides b and b divides a are true, show that a=u.b where u is a unit in R.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a If G is group, then prove the following
(i) the identity element of G is unique.
(ii) Every $a \in G$ has a unique inverse in G.

OR

b If G is a finite group and $a \in G$ then prove that
(i) 0(a) divides 0(G)
(ii) $a^{0(G)} = e$

12 a Show that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

OR

b If ϕ is a homomorphism of G onto $\overline{G}$ with Kernel K, then prove that the set of all inverse in ages $\overline{g} \in \overline{G}$ under ϕ in G is given by K_x where x is any particular inverse image of $\overline{g}$ in G.

Cont...

13 a Show that S_n has as a normal subgroup of index 2, the alternating group. An consisting of all even permutations.

OR

b If G is a group, then prove that A(G), the set of automorphisms of G is also a group.

14 a If R is a commutative ring and $a \in R$, then show that $aR = \{a.r / r \in R\}$ is a two sided ideal of R.

OR

b Prove that a finite integral domain is a field.

15 a Let R be a Euclidean ring. Then prove that every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

OR

b If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 State and prove Lagrange's theorem for finite groups.

17 Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K. Then prove that $G/K \approx \overline{G}$.

18 If I(G) is the group of inner automorphisms of G and Z is the center of G, prove that $I(G) \approx G/Z$

19 If U is an ideal of the ring R, then prove that $\frac{R}{U}$ is a ring and is a homomorphic image of R.

20 State and prove unique factorization theorem for a Euclidean ring.

Z-Z-Z END