18MAP02

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch –**MATHEMATICS**

REAL ANALYSIS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

Choose the best answer :

1 L(P, f, α) _____ L (P*, f, α)
(i) < (ii) ≤ (iii) > (iv) ≥

2 If $f_1, f_2 \in \mathscr{R}(\alpha)$, then which of the following is true.
(i) $f_1 + f_2 \in \mathscr{R}(\alpha)$ (ii) $f_1 - f_2 \in \mathscr{R}(\alpha)$
(iii) $f_1, f_2 \in \mathscr{R}(\alpha)$ (iv) all of the above

3 Find the value of $\lim_{n\to\infty} \frac{\sin nx}{\sqrt{n}}$, (x real, n = 1, 2, 3, …)
(i) 0 (ii) 0.5 (iii) 1 (iv) ∞

4 What is the value of $\lim_{n\to\infty} f_n\left(\frac{1}{n}\right)$, where $f_n(x) = \frac{x^2}{x^2 + (1-nx)^2}$?
(i) 0 (ii) 0.5
(iii) 1 (iv) ∞

5 $E(Z + 2\pi i)$ = ____________.
(i) E(Z) (ii) E(Z) +1 (iii) E (Z) - 1 (iv) 1

6 Write the value of $\Gamma(n+1)$.
(i) n! (ii) $n\Gamma(n)$
(iii) neither (i) or (ii) (iv) both (i) and (ii)

7 $|Ax|$ ________________ $\|A\||x|$.
(i) < (ii) ≤ (iii) > (iv) ≥

8 If $f'(x) = 0$ for all $x \in E$, then f is ____________.
(i) constant (ii) polynomial (iii) quadratic (iv) linear

9 The function f is said to be measurable if the set _________ is measurable for every real a.
(i) $\{x / f(x) > a\}$ (ii) $\{x / f(x) < a\}$
(iii) $\{a / f(x) \geq a\}$ (iv) all of the above

10 If f and g are measurable real valued functions on X, then ________ is measurable.
(i) $|f|$ (ii) $f + g$ (iii) f g (iv) all of the above

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Show that U(P, f, α) ≥ U (P*, f, α), where P* is a refinement of P.
OR
b Describe the term rectifiable.

Cont…

12 a Show that the limit of the integral need not be equal to the integral of the limit, even if both are finite.

OR

b If $\{f_n\}$ is a point wise bounded sequence of complex functions on a countable set E, then show that $\{f_n\}$ has a subsequence $\{f_{nk}\}$ such that $f_{nk}(x)\}$ converges for every $x \in E$.

13 a Analyse whether E (it) $\neq 1$, if $0 < t < 2\pi$.

OR

b Bring out the result $\lim\limits_{n\to\infty} S_N(f;x) = f(x)$, if for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x+t) - f(x)| \leq M|t|$ for all $t \in (-\delta, \delta)$.

14 a Let Ω be the set of all invertible linear operators on R^n. If $A \in \Omega$, $B \in L(R^n)$ and $\|B - A\| \, \|A\| < 1$, then show that $B \in \Omega$.

OR

b Suppose f maps an open set $E \subset R^n$ into R^m and f is differentiable at a point $x \in E$. Show that the partial derivative (D_jF_i) (x) exist and $f'(x)e_j = \sum_{i=1}^{m} (D_j f_i)(x) u_i, (1 \leq j \leq n)$.

15 a If $f \in \mathcal{L}(\mu)$ on E, then show that $|f| \in \mathcal{L}(\mu)$ on E and $\left| \int_E f d\mu \right| \leq \int_E |f| d\mu$.

OR

b Suppose $E \in m$. If $\{f_n\}$ be a sequence of non negative measurable functions and $f(x) = \lim\limits_{n\to\infty}$ inf $f_n(x)$ $(x \in E)$, show that $\int_E f d\mu \leq \liminf\limits_{n\to\infty} \int_E f_n d\mu$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Suppose $f \in R(\alpha)$ on [a, b], $m \leq f \leq M$, ϕ is continuous on [m, M] and $h(x) = \phi(f(x))$ on [a, b]. Show that $h \in R(\alpha)$ on [a, b].

OR

b Assume α increasing motonoically and $\alpha' \in R(\alpha)$ on [a, b]. Let f be a bounded real valued function on [a, b]. Then show that $f \in R(\alpha)$ if and only is $f\alpha' \in R(\alpha)$.

Also show that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x) dx$.

17 a Let α be monotonically increasing on [a, b]. Suppose $f_n \in R(\alpha)$ on [a, b], for n = 1, 2, ,3...and suppose $f_n \to f$ uniformly on [a, b]. Then show that $f \in R(\alpha)$ on [a, b] and $\int_a^b f d\alpha = \lim\limits_{n\to\infty} \int_a^b f_n d\alpha$.

OR

b Analyse the property that there exists a real continuous function on the real line which in nowhere differentiable.

18 a Suppose $\sum C_n$ converges and let $f(x) = \sum_{n=0}^{\infty} C_n x^n (-1 < x < 1)$, then show that $\lim\limits_{x\to 1} f(x) = \sum_{n=0}^{\infty} C_n$.

OR

b State and prove Parseval's theorem.

19 a State and prove the inverse function theorem.

OR

b State and prove the implicit function theorem.

20 a Elucidate $m(\mu)$ as a σ-ring and μ^* is countably additive on $m(\mu)$.