14MAP08

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(Second Semester)

Branch –**MATHEMATICS**

MATHEMATICAL STATISTICS

Time: Three Hours Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a The random variable X has the normal distribution with density $f(x)=\left(\frac{1}{\sqrt{2\pi}}\right)e^{-x^2/2}$. Find $E(x^2)$. (5)

b State and prove Chebyshev inequality. (6)

c Prove that the characteristic function of the sum of an arbitrary finite number of independent random variables equal the product of their characteristics functions. (4)

OR

d The joint distribution of the random variable (X Y) is given by the density $f(x, y)=\begin{cases}\frac{1}{4}[1+xy(x^2-y^2)] & for\ |x|\le 1\ and\ |y|\le 1\\ 0 & for\ all\ other\ points\end{cases}$

Show that the random variables X and Y are dependent. Also find the density function of the sum Z = X + Y. (8)

e If for a random variable X the absolute moment of order n exists, for arbitrary K (K = 1, 2, , ….n-1) then prove that the following inequality is true : $\beta_k^{1/k} \le \beta_{k+1}^{1/k+1}$. (7)

2 a Define uniform distribution. Find μ_2 of the uniform distribution. (6)

b The random variable X has the distribution N (1, 2). Find the probability that X is greater than 3 is absolute value. (4)

c State and prove addition theorem of gamma distribution. (5)

OR

d Let the random variable X_n have a binomial distribution defined by the formula $P(X_n=r)=\frac{n!}{r!(n-r)!}p^r(1-p)^{n-r}$, where r takes on the values 0, 1, 2,….n…If for n =1 , 2, … the relation $p=\frac{\lambda}{n}$ holds where $\lambda>0$ is a constant, prove that $\lim_{n\to\infty}P(X_n=r)=\frac{\lambda^r}{r!}e^{-\lambda}$. (6)

e The random variable X has the gamma distribution with the density

$f(x)=\begin{cases}0 & for\ x\le 0\\ 2e^{-2x} & for\ x>0\end{cases}$, what is the probability that X is not smaller than

3 a Prove that the sequence of random variable $\{X_n\}$ given by

$$p\left\{Y_n = \frac{r}{n}\right\} = \binom{n}{r} p^r (1-p)^{n-r}$$, where $0 < p < 1$ and $r = 0, 1, 2, \ldots n$ and

$X_n = Y_n - p$ is stochastically convergent to 0, ie for any $\in > 0$ we have $\lim_{n\to\infty} P(|X_n| > \in) = 0$. (8)

b State and prove Lindeberg – Levy theorem. (7)

OR

c State and prove the De Moivre Laplace theorem. (15)

4 a Prove that a stochastic process $\{X_t, 0 \le t \le \infty\}$ where X_t is the number of signals in the interval [0, t], satisfying conditions.

i) The process $\{X_t, 0 \le t \le \infty\}$ is a process with independent increments.

ii) The process $\{X_t, 0 \le t \le \infty\}$ is a process with homogenous increments.

iii) The following relations are satisfied $\lim_{t\to 0} \frac{W_1(t)}{t} = \lambda (\lambda > 0)$:

$\lim_{t\to 0} \frac{1 - W_0(t) - W_1(t)}{t} = 0$ and the equality $P(X_0=0)=1$ is a homogenous poisson process. (15)

OR

b State and prove Furry – Yule process. (7)

c Prove that a function R(T) is the correlation function of a process $\{Z_t, -\infty < t < \infty\}$ stationary in the wide sense, continuous and satisfying $m = 0$, $\sigma = 1$, if and only if there exists a distribution function $F(\lambda)$ such that $R(T) = \int_{-\infty}^{\infty} e^{i\lambda r} dF(\lambda)$. (8)

5 a Define χ^2 distribution and derive the density function of χ^2 distribution with one and six degrees of freedom. (15)

OR

b Obtain the distribution of the two dimensional random variables $(\overline{X}, S)$, where $\overline{X} = \frac{1}{n}\sum_{k-1}^{n} X_k$; $S^2 = \frac{1}{n}\sum_{k=1}^{n}(X_k - \overline{X})^2$. (15)

Z-Z-Z END