14MAP09

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(Third Semester)

Branch –**MATHEMATICS**

FUNCTIONAL ANALYSIS

Time: Three Hours Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Prove that every Cauchy sequence is bounded. (3)

b Define Isometric isomorphism of a normed spaces. (3)

c Let X be a closed subspace of a normed space Y. Then prove that the factor (or) quotient space Y / X is a normed space under the norm $\|y+x\| = \inf\{\|y+x\| : x \in X\}$. (9)

OR

d Let X and Y be normed spaces and T a linear operator on X into Y. Then prove that the following statements are equivalent :
(i) T is continuous
(ii) T is continuous at the origin
(iii) T is bounded
(iv) TS_1 is a bounded subset of Y (8)

e If X is Banach space and $T \in \mathcal{B}(X)$ such that $\|T\| < 1$, then prove that I – T is invertible. (5)

f Define a Banach Algebra. (2)

2 a Define an inner product space. (3)

b Prove that every inner product space X is a normed space with respect to the norm $\|x\| = |< x, x >|^{1/2}$, $\forall x \in X$. (6)

c Prove that if M and N are closed subspace of a Hilbert space X such that $M \perp N$, the subspace $M + N = \{x + y \in X ; \quad x \in M$ and $y \in N\}$ is also closed. (6)

OR

d State and prove the Bessel's inequality. (8)

e Prove that every Hilbert space X is reflexive. (7)

3 a Prove that an operator T* is bounded, linear and unique. (6)

b Prove that the proper values of a self-adjoint operator are real numbers and two proper vectors corresponding to two different proper values of a self-adjoint operator are orthogonal. (5)

c Prove that a bounded linear operator T on a Hilbert space X is normal iff $\|T^* x\| = \|Tx\|$ for every $x \in X$. (4)

OR

d Define sesqulinear function. (3)

e State and prove the Lax-Milgram Lemma. (10)

c State Banch – Steinhaus theorem. (3)

OR

d State and prove closed graph theorem. (8)

e Let $\{T_n\}$ be a sequence of continuous linear operators of Banach space X into Banach space Y that $\lim_{n\to\infty} T_n x = Tx$ exists for every $x \in X$. Then, prove that T is continuous linear operator and $\|T\| \le \liminf_{n\to\infty} \|T_n\|$. (7)

5 a State and prove Banach Contraction Principle. (10)

b State Schauder's Fixed – Point theorem. (3)

c Define a contraction mapping. (2)

OR

d Let T be an operator on X. Then prove that S and TST^{-1} have the same eigen values. (3)

e Prove that the adjoint of a compact operator is compact. (5)

f Let T be an non-zero, compact, self-adjoint operator on a Hilbert space. Then prove that T has an eigen value λ equal to either $\|T\| or -\|T\|$. (7)s

Z-Z-Z END