14MAP07

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(Second Semester)

Branch –**MATHEMATICS**

TOPOLOGY

Time: Three Hours Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Let B and B' be bases for the topologies T and T', respectively , on X. Then prove that the following are equivalent :
i) T' is finer than J
ii) For each $x \in X$ and each basis element $B \in$ B containing x, there is a basis element $B' \in B'$ such that $x \in B' \subset B$.

OR

b Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

2 a Let $f : A \to$ X x Y be given by the equation f(a) = (f_1(a), f_2 (a)). Then prove that f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous.

OR

b State and prove uniform limit theorem.

3 a i) Prove that the image of a connected space under a continuous map is connected.

ii) Prove that every closed subspace of a compact space is compact.

OR

b Prove that a subspace A of R^h is compact if and only is it is closed and is bounded in the Euclidean metric d or the square metric p.

4 a Let X be a metrizable space. Then prove that the following are equivalent :
(i) X is compact
(ii) X is limit point compact
(iii) X is sequentially compact.

OR

b Prove that every regular space with a countable basis is normal.

5 a State and prove Urysohn lemma.

OR

b State and prove Tychonoff theorem.

Z-Z-Z END