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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Third Semester)

Branch –**MATHEMATICS**

CORE ELECTIVE-I STOCHASTIC DIFFERENTIAL EQUATION

Time: Three Hours Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Prove that B_t is a Gaussian process.

OR

b Define $\Delta B_t = B_{tk+s} - B_{t_k}$ and put $Y(t,w) = \sum_{t_k st} (\Delta B_k(w))^2$, show that $E\left[\left(\sum(\Delta B_k)^2 - t\right)^2\right] = 2\sum_{t_k st}(\Delta t_k)^2$ and deduce that $Y(t,.) \to t$ in $L^2(P)$ as $\Delta t_k \to \infty$.

c Define $Y : \Omega \to R$ by $Y(1) = 0$, $Y(2) = Y(3) = Y(4) = Y(5) = 1$. Find the σ- algebra $\mathcal{H}_y$ generated by Y.

2 a State and proved the Ito isometry.

OR

b Let $f \in \gamma(0,T)$. Then there exists a t-continuous version of $\int_0^t f(s,w)dB_s(w); 0 \le \in \le T$, ie there exists a t-continuous stochastic process J_t on (Ω, F, P)s, t $P\left[J_t = \int_0^t fdB\right] = 1 \forall t,\ 0 \le t \le T$.

3 a State and prove the 1-dimensional Ito formula.

OR

b State the prove the existence ad uniqueness theorem for stochastic differential equations.

4 a Obtain the solution of the filtering problem ie, a stochastic differential equation for X_t.

OR

b Find the Noisy observation of a Brownian motion.

5 a State and prove the Markov property for Ito diffusions.

OR

b State and prove Dynkin's formula.

Z-Z-Z END