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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Fourth Semester)

Branch –**MATHEMATICS**

CONTROL THEORY

Time : Three Hours Maximum : 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Let A(t) be an n x n matrix that is continuous on a closed bounded interval J and let $f \in L^2{}_n(J)$. Given $t_o \in J$ and $x_o \in R^n$, then prove that there exists a unique solution x(t) of $\dot{x}(t) = A(t)x(t) + f(t)$ on the interval J with $x(t_0) = x_0$. (15)

OR

b i) Prove that the observed linear system $\dot{x}(t) = A(t)x(t)$ and $y(t) = H(t)x(t)$ is observable on [0, T] iff the observability Grammian matrix W(0, T) = $\int_0^T X^*(t,0)H^*(t)H(t)X(t,o)dt$ is positive definite, where the * denotes the transpose of the matrix. (8)

ii) Prove that there exist a reconstruction Kernel R(t) on [0,T] iff the observed system $\dot{x}(t) = A(t)x(t)$ and y(t) = H(t)x(t) is observable on [0, T]. (7)

2 a Prove that the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ is controllable on [0,T] iff the adjoint linear system $\dot{y}(t) = -A^*(t)y$ and w(t)=B*(t)y is observable on [0, T]. (15)

OR

b Determine the control function for the controlled harmonic oscillator $\ddot{x} + x = u$ which stears from $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$. (15)

3 a i) Prove that the system $\dot{x}(t) = A(t)x(t)$ is uniformly asymptotically stable iff the exist constants $\alpha > 0, K > 0$ with $\|X(t,s)\| \le Ke^{-\alpha(t-s)}, 0 \le s, t < \infty$. (8)

ii) Consider the system $\dot{x} = Ax(t)$, where $A = \begin{pmatrix} -1 & 0 & 0 \\ -2 & -1 & 2 \\ -3 & -2 & -1 \end{pmatrix}$. Show that the given system is stable. (7)

OR

b i) State and prove Gronwalls' inequality. (8)

ii) If all the characteristic roots of A have negative real parts and B(t) satisfies $\lim_{t\to\infty} \|B(t)\| = 0$, then prove that all the solutions of the system $\dot{x}(t) = Ax(t) + B(t)x(t)$ tends to zero as $t \to \infty$. (7)

4 a Stabilize the system $\ddot{x} - x = u$ by Bass's method. (15)

OR

b Prove that the control problem $x(0) = x_0$, $x(T) = x_1$ for the system $\dot{x} = Ax + Bu$ is solvable iff $x_1 - e^{AT}x_0 \in C(A,B)$. (15)

5 a Given the linear system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ and the cost functional $J = \frac{1}{2} x^*(T)\, Fx(T) + \frac{1}{2} \int_0^T [x^*(t)Q(t)x(t) + u^*(t)R(t)u(t)]dt$, prove that there exists an optimal control of the form $u(t) = -R^{-1}(t)B^*(t)K(t)x(t)$ where $K(t)$ is the solution of the Riccati equation with $K(T) = F$. (15)

OR

b Obtain the optimal control for the controllable system $\dot{x}_1(t) = x_2(t), \dot{x}_2(t) = u(t)$ with the cost functional $J = \frac{1}{2} \int_0^\infty \left[x_1^2(t) + 2bx_1(t)x_2(t) + ax_2^2(t) + u^2(t)\right]dt$. where we assume that $a - b^2 > 0$. (15)

Z-Z-Z END