

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Second Semester)

Branch – MATHEMATICS

COMPLEX ANALYSIS

Time : Three Hours

Maximum : 75 Marks

Answer ALL questions
ALL questions carry EQUAL marks (5 x 15 = 75)

- 1 a Define Cross Ratio. If z_1, z_2, z_3, z_4 are distinct points in the extended plane and T any linear transformation, then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$. (7)
- b State and prove Cauchy's theorem in a disk. (8)
- OR
- c If the piecewise differentiable closed curve γ does not pass through the point a , then prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$. (7)
- d State and Prove Cauchy's theorem for a Rectangle. (8)
- 2 a State and prove Residue theorem. (7)
- b State and prove Rouché's theorem. (8)
- OR
- c State and prove Schwartz's theorem. (7)
- d Find the poles and residues of the following functions. (8)
- (i) $\frac{1}{z^2 + 5z + 6}$ (ii) $\cot z$
- 3 a State and prove Weierstrass's theorem. (7)
- b State and prove Taylor's series. (8)
- OR
- c Define infinite product. Show that the necessary and sufficient condition for the absolute convergence of the product $\prod_1^{\infty} (1 + a_n)$ is the convergence of the series $\sum_1^{\infty} |a_n|$. (8)
- d State and prove Poisson – Jensen's formula. (7)
- 4 a State and prove Schwartz-Christoffel's formula. (10)
- b Discuss the concept of Boundary Behaviour. (5)
- OR
- c State and prove Harnack's principle. (7)

4 Cont...

d Prove that the continuous function $u(z)$ which satisfies

$$u(z_0) = \frac{1}{2\pi} \int_0^{2\pi} u(z_0 + re^{i\theta}) d\theta \text{ is necessarily Harmonic.} \quad (8)$$

5 a Prove that any two bases of the same module are connected by a unimodular transformation. (7)

b Derive relationship between $\zeta(z)$ and $\sigma(z)$. (8)

OR

c Derive the Weierstrass \wp -function. (8)

d Prove that the sum of the residues of an elliptic function is zero. (7)

Z-Z-Z

END