PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Second Semester)

Branch –**MATHEMATICS**

TOPOLOGY

Time : Three Hours   Maximum : 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks   (5 x 15 = 75)

1 a i) Define a topology. (3)

ii) Define a product topology. If $\mathscr{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then prove that the collection $D = \{B \times C \,/\, B \in \mathscr{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of X x Y. (12)

OR

b i) Define a Hausdorff space. (3)

ii) Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. (12)

2 a i) Let X and Y be topological spaces; let $f : X \to Y$. Then prove that the following are equivalent :

(i) f is continuous

(ii) for every subset A of X, one has $f(\overline{A}) \subset \overline{f(A)}$

(iii) for every closed set B of Y, then set $f^{-1}(B)$ is closed in X

(iv) for each $x \in X$ and each neighborhood V of f(x), there is a neighborhood U of x such that $f(U) \subset V$. (15)

OR

ii) Let X be a metric space with metric d. Define $\overline{d} : X \times X \to R$ by the equation $\overline{d}(x, y) = \min\{d(x, y), 1\}$. Then prove that $\overline{d}$ is a metric that induces the same topology as d. (8)

iii) State and prove uniform limit theorem. (7)

3 a i) Prove that the image of a connected space under a continuous map is connected. (8)

ii) Prove that a finite Cartesian product of connected spaces is connected. (7)

OR

b i) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X. (7)

ii) Prove that every compact subspace of a Hausdorff space is closed. (8)

4 a Let X be a metrizable space. then prove that the following are equivalent :

(i) X is compact

(ii) X is limit point compact

(iii) X is sequentially compact (15)

OR

b i) Prove that every compact Hausdorff space is normal. (8)

ii) Prove that every well-ordered set X is normal in the order topology. (7)

5 a State and prove Urysohn lemma. (15)

OR

b State and prove Tychonoff theorem. (15)

Z-Z-Z   END