14MAP02

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(First Semester)

Branch --**MATHEMATICS**

REAL ANALYSIS

Time: Three Hours Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Assume α increases monotonically and on [a, b] and $\alpha \in R$ on [a, b]. Let f be bounded real function on [a, b], then $f \in R(\alpha)$ on [a, b]. In this case prove that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$. (8)

b State and prove Fundamental theorem of calculus. (7)

OR

c Prove that $f \in R(\alpha)$ on [a, b] iff for every $\varepsilon > 0$ there exists a partition P such that $U(p, f, \alpha) - L(p, f, \alpha) < \alpha$. (7)

d If $f, g \in R(\alpha)$ on [a, b], then prove that (i) $fg \in R(\alpha)$ (ii) $|f| \in R(\alpha)$ and (iii) $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$. (8)

2 a State and prove Cauchy Criterion for uniform convergence. (8)

b Prove that there exists a real continuous function on the real line which is nowhere differentiable. (7)

OR

c If K is compact, if $f_n \in \mathcal{C}(K)$ for n = 1, 2, 3.....and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, prove that
(i) $\{f_n\}$ is uniformly bounded on K
(ii) $\{f_n\}$ contains a uniformly convergent subsequence. (15)

3 a If x > 0 and y > 0, then prove that $\int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$. (7)

b Suppose a_0, a_1,...a_n are complex numbers, $n \geq 1$, $a_n \neq 0$, $P(z) = \sum_0^n a_k z^k$, then prove that P(z)= 0 for some complex number z. (8)

OR

c State and prove Parsevel's theorem. (15)

4 a State the prove Contraction Principle theorem. (8)

b Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then prove that dim $X \leq r$. (7)

OR

c State and prove Inverse function theorem. (15)

5 a State and prove Implicit function theorem. (15)

OR

b If [A] and [B] are n by n matrices, then prove that det([B][A])=det[A] det[B]. (8)

c Prove that a linear operator A on R^n is invertible iff det[A] $\neq 0$. (7)