14MAP06

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Second Semester)

Branch –**MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATIONS

Time : Three Hours | Maximum : 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Find the general integral of the linear partial differential equation $(y + 2x)p - (x + yz)q = x^2 - y^2$. (6)

b Find the general integral of the equation $(x - y)p + (y - x - z)q = z$ and the particular solution through the circle $z = 1;\ x^2 + y^2 = 1$. (9)

OR

c Show that the equation $xp = yq;\ z(xp + yq) = 2xy$ are compatible and solve them. (8)

d Find the complete integrals of the equation $p = (z + qy)^2$. (7)

2 a If $u = f(x + iy) + g(x - iy)$ where the functions f and g are arbitrary, then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. (3)

b Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form. (12)

OR

c Find a particular integral $(D^2 - D^1)z = e^{x+y}$. (7)

d Solve the one dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$. (8)

3 a If $\rho > 0$ and $\psi(\bar{r}) = \int_r \frac{\rho(\bar{r}')dT'}{|\bar{r} - \bar{r}'|}$ where the volume V is bounded, then prove that $\lim_{r=\infty} \bar{r}\psi(\bar{r}) = M$, where $M = \int_V \rho(\bar{r}')dT'$. (10)

b State (i) Interior Newmann problem, (ii) Exterior Newmann problem. (3+2)

OR

c A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undistributed state is V. Determine the velocity of the fluid at any point of the distributed system. (15)

Cont…

4 a Write a short notes on Longitudinal vibrations in a Bar. (7)

b Derive D' Alembert's solution of one-dimensional wave equation. (8)

OR

c Find approximate values for the first three eigen values of a square membrane of side 2. (15)

5 a State and prove Duhamel's theorem. (15)

OR

b Show that the function $\theta = \frac{1}{\sqrt{t}} \exp\left(-\frac{x^2}{4kt}\right)$ is a solution of the equation

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{k}\frac{\partial \theta}{\partial t}.$$ (5)

c Determine the temperature $\theta(\rho,t)$ in the infinite cylinder $0 \le \rho \le a$ when the initial temperature is $\theta(\rho,0) = f(\rho)$ and the surface $\rho = a$ is maintained at zero temperature. (10)

Z-Z-Z END