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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Fourth Semester)

Branch –**MATHEMATICS**

OPERATOR THEORY

Time : Three Hours | Maximum : 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Prove that for any bounded linear operator T, $\|T\| = \sup\{|(Tx, y)| : \|x\| = \|y\| = 1\}$. (5)

b Let T be an operator on a Hilbert space H. Then show that T* is also an operator on H and the following properties hold :
(i) $\|T^*\| = \|T\|$ (ii) $(T_1 + T_2)^* = T_1^* + T_2^*$
(iii) $(\alpha T)^* = \overline{\alpha} T^*$ for any $\alpha \in C$ (iv) $(T^*)^* = T$ (v) $(ST)^* = T^*S^*$ (10)

OR

c Show that for any positive operator A, there exists the unique positive operator S such that $S^2 = A$ and $(S) \supset (A)$ (10)

d State and prove the Generalised Schwarz inequality. (5)

2 a Let M be a dense subspace of a normed space X. Let T be a linear operator from M to a Banach space Y. If T is bounded, then show that there uniquely exists $\overline{T}$ which is the extension of T from X to Y. (10)

b Let $T = U|T|$ be the polar decomposition of an operator T on a Hilbert space H. Prove that $T^* = U^*|T^*|$ is also the polar decomposition of an operator T*. (5)

OR

c If T = UP is the polar decomposition of an operator T, then prove that U and P commutes with A and A* where A denotes any operator which commutes with T and T*. (8)

d State and prove the Fuglede – Putnam theorem. (7)

3 a Prove
(i) If T is an operator such that $\|I - T\| < 1$, then T is invertible. (5)
(ii) If an operator T is normal, then $\sigma(T) = A_\sigma(T)$. (5)
(iii) ½ $\|T\| \le w(T) \le \|T\|$ for any operator T. (5)

OR

b State and prove
(i) Spectral mapping theorem (5)
(ii) Toeplitz – Hausdorff theorem. (10)

Cont...

4 a Prove the following inclusion relations :
Self adjoint $\subseteq$ Normal $\subseteq$ Quasinormal $\subseteq$ Subnormal $\subseteq$ Hyponormal $\subseteq$ Paranormal $\subseteq$ Normaloid $\subseteq$ Spectraloid (15)

OR

b State and prove
(i) Holder – McCarthy inequality (10)
(ii) Young inequality (5)

5 a Let $T = U|T|$ be the polar decomposition of a log-hyponromal operator. the prove that $\widetilde{T}_{s,t} = |T|^s U|T|^t$ is $\frac{\min\{s,t\}}{s+t}$ - hyponromal for any $s > 0$ and $t > 0$.. (15)

OR

b Prove
(i) Every log-hyponormal operator is a class A operator. (4)
(ii) Every class A operator is a paranormal operator. (4)

c Give an example of a non-class A, class A(2) and paranormal operator. (7)

Z-Z-Z END