14MAP08

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(Second Semester)

Branch –**MATHEMATICS**

MATHEMATICAL STATISTICS

Time : Three Hours | Maximum : 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a i) Define the standardized random variable. Also, obtain its mean and variance. (4)

ii) Define Symmetric distribution in terms of its distribution function as well as density function. (3)

b Obtain the expression for density function of a random variable, from that of its characteristic function. (8)

OR

c $g_1(X)$ and $g_2(X)$ are two single-valued function of a random variable X and the expected values $E[g_1(X)]$ and $E[g_2(X)]$ exist. Prove that $E[(g_1(X) + g_2(X)] = E[g_1(X)] + E[g_2(X)]$. (5)

d Define variance of a random variable X. Obtain the variance of $Y = aX+b$. (5)

e Obtain the density function of the random variable X, whose characteristic function is $\varphi(t) = e^{-\frac{t^2}{2}}$. (5)

2 a Write a detailed note on : 'One-point' distribution. (7)

b Define a normal random variable. Obtain the expressions for the moments of normal distribution. (8)

OR

c Derive an expression for the characteristic function of 'Uniform Distribution' and hence its mean and variance. (8)

d Derive the mean and variance of beta distribution. (7)

3 a Define 'Stochastic Convergence'. Explain with an example. (7)

b State and prove 'Lindeberg – Levy' theorem. (8)

OR

c State and prove the sufficient part of Levy – Cramer theorem. (9)

d A coin is thrown 100 times. The number 1 is assigned to the appearance of heads and the number 0 to that of tails. Using De Moivre – Laplace theorem, find the probability that heads will appear more than 50 times and less than 60 times? (6)

Cont...

4 a i) Define 'Markov Process'. (3)

ii) State the condition under which a markov process becomes homogeneous. (2)

iii) State the condition under which a markov process becomes a process with independent increments. (2)

b Obtain the system of differential equations under the Furry-Yule process, and obtain its solution. (8)

OR

c State and prove the conditions under which a stochastic process becomes a homogenous Poisson process. (15)

5 a Given $\bar{x}$ is the sample mean of a normal distribution $N(m, \sigma)$, find its distribution. (8)

b A simple sample of size 12 is drawn from a population in which the characteristic X has N(1; 2). the observed values are : 2, 2, 5, 0.5, 1, 0, -0.9, 5.1, -1.5, 0.8, 1.1, 0.8, 0.4. Given the statistic $z = ns^2$, where s – is sample standard deviation, what is the probability that z will exceed or equals 32.28? (7)

OR

c Define χ^2 statistic and derive its density. (7)

d Obtain the asymptotic distribution of student's 't' distribution. (8)

Z-Z-Z END