14MAP01

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2018
(First Semester)

Branch –**MATHEMATICS**

ALGEBRA

Time: Three Hours　　　　　　　　　　Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks　　　　(5 x 15 = 75)

1 a If o(G) where p is a prime number, then prove that $Z(G) \neq (e)$. (9)

b If $o(G) = p^2$ where p is a prime number, then prove that G is abelian. (6)

OR

c Prove that $n(k) = 1 + p + ... + p^{k-1}$. (5)

d Prove that the number of p-Sylow subgroups in G, for a given prime, is of the form $1 + kp$. (10)

2 a If f(x), g(x) are non-zero elements of F[x], then prove that $\deg(f(x) \leq \deg f(x)\, g(x)$. (10)

b State and prove the division Algorithm. (5)

OR

c State and prove the Eisenstein criteria. (6)

d If R is a unique factorization domain, then prove that so in R[x]. (9)

3 a The element $a \in K$ is algebraic over F iff F(a) is a finite extension of F. (15)

OR

b Prove that a polynomial of degree n over a field can have at most n roots in any extension field. (15)

4 a For any $f(x).g(x) \in F[x]$ and any $a \in F$. Then prove that
(i) $(f(x) + g(x)) = f(x) + g(x)$　(ii) $(\alpha f(x)) = \alpha f(x)$
(iii) $(f(x)g(x)) = f(x)g(x) + g(x)f(x)$ (10)

b Prove that any finite extension of a field of characteristic 0 is a simple extension. (5)

OR

c If K is a finite extension of F, then prove that G(K, F) is a finite group and its order, o(G(K,,F)) satisfies $o(G(K, F)) \leq [K : F]$. (8)

d Define normal extension of F. (2)

e If K is a normal extension of F iff K is a splitting field of some polynomial over F. (5)

5 a For A, B $\in F_n$ and $\lambda \in F$, prove that
(i) $tr(\lambda A) = \lambda\, trA$　(ii) $tr(A + B) = tr\,A + tr\,B$　(iii) $tr\,(AB) = tr\,(BA)$ (8)

b For A, B $\in F_n$, prove that
(i) $(A')' = A$　(ii) $(A + B) + A' + B'$　(iii) $(AB)' = B'A'$ (7)

OR

c If $(vT, vT) = (v, v)$ for all $v \in V$, then prove that T is unitary. (5)

d If $T \in A(V)$ is Hermitian, then prove that all its characteristic roots are real. (5)

e If N is normal and AN = NA, then prove that AN* = N*A. (5)