

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch - STATISTICS

PROBABILITY THEORY

Time: Three Hours

Maximum: 75 Marks

Answer ALL questions
ALL questions carry EQUAL marks (5 x 15 = 75)

1 a) If $Y_n, Z_n, \lim x_n, \lim x_n$ are extended random variables, then prove that $\lim x_n$ is also an extended r.v, when it exists.

OR

- b i) If $E|x|^r < \infty$, then prove that $E|x|^s < \infty$ for $0 < s < r$ and assume that $E x^k$ exists and is finite for $k < r$, k is an integer.,
ii) State and prove Holder's Inequality.

2 a i) State Bodmer's theorem.

ii) Find the probability function to the characteristic function

$$Q(t) = \begin{cases} -|t|, & |t| < 1 \\ 0, & t > 1 \end{cases}$$

OR

b i) State and prove the Levy continuity theorem,

- ii) For a distribution, $K_r = n > 0$, find the characteristic function.
Here K_r is the r^{th} cumulant.

3 a i) Define class of Independent events.

ii) Prove that subclasses of Independent classes are independent.

iii) Let $P(W_i) = X$, $i=1,2,3,4$. If $A=\{W_1, W_2\}$, $B=\{W_1, W_3\}$ and $C=\{W_1, W_4\}$, then show that A,B and C are pairwise independent but not mutual independent.

OR

b i) State and prove the Borel-Cenelli lemma.

- ii) If x_j, s are independent and $X_n \rightarrow 0$ (a.s) then prove that $X^n [|X_n|^{>c}] < G^0 >$ whatever be $c > 0$.

4 a i) If $x_n \rightarrow x$ and $x_n \rightarrow x^1$, then show that x and x^1 are equivalent.

ii) Prove that " $x_n \rightarrow 0$ iff $E(\frac{|x_n|}{n}) \rightarrow 0$ as $n \rightarrow \infty$."

OR

b i) Define "Convergence Almost Surely".

- ii) Prove that "A sequence of random variables converges almost surely to a random variable iff the sequence converges mutually a.s".

5 a i) State and prove the Kolmogorov Inequality.

OR

b) State and prove the Kolmogorov SLLN for iid random variables.