18STP02

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch – **STATISTICS**

PROBABILITY THEORY

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 If the two events A and B are such that $A \subset B$, identify the relation between P(A) and P(B) is
(i) $P(A) \leq P(B)$ (ii) $P(A) \geq P(B)$
(iii) $P(A) = P(B)$ (iv) $P(A) / P(B)$

2 If F(x) is a distribution function of a random variable X then $F(-\infty)$ and $F(-\infty)$ is
(i) 0, 1 (ii) 1, 0
(iii) -1,0 (iv) 0, -1

3 If $\Phi_x(t)$ is the characteristic function of X, what is the value of $\Phi(0)$
(i) 0 (ii) 1
(iii) Φ (iv) none of these

4 If $U = \dfrac{X-a}{h}$, a and h are constants find $\Phi_U(t)$
(i) $e^{(\iota at / h)}\varphi_x(t/h)$ (ii) $e^{(-\iota at/h)}\varphi_x(-t/h)$
(iii) $e^{(-\iota at/h)}\varphi_x(t/h)$ (iv) $e^{(-\iota at/h)}\varphi_x(t)$

5 Two random variables X and Y are said to be independent if
(i) E(XY) = 1 (ii) E(XY) = 0
(iii) E(XY) = E(X)E(Y) (iv) E(XY) = a constant

6 In Borel 0 – 1 law, if $\sum_{n=1}^{\infty} P(A_n) < \infty$, Indicate the value of P(A).
(i) 0 (ii) 1
(iii) ∞ (iv) $-\infty$

7 X_n converges to X almost surely, $X_n \to X$, if there is a (measurable) set $A \subset \Omega$ such that P(A) =
(i) 0 (ii) 1
(iii) -1 (iv) ∞

8 If $X_n \xrightarrow{p} 0$, if $E|X_n| \xrightarrow{r} ?$
(i) 0 (ii) 1 (iii) -1 (iv) ∞

9 In Lindeberg Levy theorem the assumption of variables is
(i) not independent but identically distributed
(ii) independent and identically distributed
(iii) independent but not identically distributed
(iv) not independent and not identically distributed

10 If the variables are uniformly bounded the necessary and sufficient condition for WLLN to hold is

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a State and prove R inequality.

OR

b State and prove additional theorem of Expectation.

12 a Define characteristic function and state its properties.(any four).

OR

b State the following theorems :
(i) Uniqueness theorem of characteristic function
(ii) Khinchine – Bochner's theorem

13 a State and prove Borel 0-1 law.

OR

b Write equivalent definition of independent events and random variables.

14 a Prove that $X_n \xrightarrow{r} X \Rightarrow E|X_n|^r \to E|X|^r$.

OR

b Prove that $X_n \xrightarrow{r} X \Rightarrow X_n \xrightarrow{p} X$. If Xn's are almost surely bounded.

15 a State Liaponov's central limit theorem.

OR

b State and prove Kolmogorov's strong law of large numbers.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a State and Prove Chebychev's inequality.

OR

b A random variable X has the following probability function :

x	0	1	2	3	4	5	6	7
p(X = x)	0	3k	5k	7k	11k	13k	15k	17k

(i) Determine the value of k.
(ii) Evaluate P(X < 3), P(X ≥3).
(iii) Determine the distribution function X.

17 a State and prove Levy continuity theorem.

OR

b Find the density function f(x) corresponding to characteristic function defined as follows : $\varphi(x) = \begin{cases} 1-|t|, & |t| \le 1 \\ 0, & |t| \ge 1 \end{cases}$.

18 a State and prove Kolmogorov 0-1 law.

OR

b Given that the joint p.d.f $f(x, y) = 4xye^{-(x^2+y^2}), x \ge 0, y \ge 0$, (i) find Marginal density function of x and y, (ii) Check whether the random variables are independent.

19 a Prove that a function $X_n \xrightarrow{a.s} X$, iff as $n \to \infty$., $P[\cup(w|x_k - x| \ge 1/r)] \to 0$ for every r, where r is an integer.

OR

b State and prove Helly Bray theorem.

20 a State and prove Lindeberg Levy central limit theorem.