

PSG COLLEGE OF ARTS & SCIENCE  
(AUTONOMOUS)  
BSc DEGREE EXAMINATION MAY 2017  
(First Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

CALCULUS

Time : Three Hours

\* . Maximum : 75 Marks

SECTION-A (20 Marks)

4 Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

1 When will a function  $f(x, y)$  has minimum value of  $x = a, y = b$ ?

2 State Lagrange's method.

3 Give the Cartesian formula for the radius of curvature.

4 Find  $\frac{dy}{dx}$  when  $x = a(\cos t + 1 \sin t), y = a(\sin t - 1 \cos t)$ .

$-\frac{n}{2}$

5 Prove that  $\int \sin^{11} x \, dx = \int \cos^n x \, dx$ .

6 • Evaluate  $\int \sin^6 x \, dx$

0

3 2

7 Evaluate  $\int \int xy \, dx \, dy$

0 1

a x .

8 Evaluate  $\int_0^1 \int_0^1 (x^2 - y^2) \, dx \, dy$

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9 Find  $T(1)$ .

10 Give the relation

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

11 a Give the working rule to examine a function  $u = f(x, y)$  for maxima and minima.

OR

b Discuss the maxima and minima of the function  $z = 6 - x - y$ .

12 a Show that in the parabola  $y^2 = 4ax$  at the point  $P, p = -2a(1 + 1^2)$

OR

b Find the radius of curvature of the cardioid  $r = a(1 - \cos \theta)$ .

Cont...

13 a Prove that  $\int_0^{\pi/2} \sin^3 \theta \, d\theta = \frac{2}{3}$ .

OR

b Derive the reduction formula for  $I_n = \int \sin^n x \, dx$ .

14 a Evaluate  $\iint (x^2 + y^2) \, dx \, dy$  over the region for which  $x, y$  are each  $> 0$  and  $x + y < 1$ .

OR

b Evaluate  $\int_0^1 \int_{y^2}^{1-y} x^2 \, dx \, dy$ .

15 a Express  $\int_0^1 x^m (1-x)^n \, dx$  in terms of Gamma functions.

OR

b Evaluate  $\int_0^{\pi/2} \sin^{10} \theta \, d\theta$ .

SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

16 If  $u = a^3x^2 + b^3y^2 + c^3z^2$  where  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ , find the minimum value of  $u$ .

17 Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

18 Derive the reduction formula for  $\int \sin^m x \cos^n x \, dx$ .

19 Evaluate  $\iiint xyz \, dx \, dy \, dz$  taken through the positive octant of the sphere  $x^2 + y^2 + z^2 = a^2$ .

20 Prove that  $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2 y^2 z^2 \, dz \, dy \, dx = \frac{1}{540}$ , the integration extended to all

positive values of the variables for which the expression is real.

**Z-Z-Z****END**