

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2017
(Sixth Semester)

Branch - MATHEMATICS

COMPLEX ANALYSIS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 x 2 = 20)

- 1 Define analytic a function.
- 2 Find $f(z)$ where $f(z) = z^2$.
- 3 Define critical point of $f(z)$.
- 4 What are fixed points of the mapping $f(z) = az$.
- 5 Find $\int_C \frac{dz}{z}$ where C is $|z| = 1$.
- 6 Define simply connected region.
- 7 Find all the zeros of $\cos z$.
- 8 Define singularity of $f(z)$.
- 9 What is the residue at $z = a$ for $f(z) = \frac{1}{(z-a)^m}$?
- 10 Find the residue of $\cot z$ at $z = 0$.

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Prove that $f(z) = \operatorname{Re} z$ are nowhere differentiable.
OR
b Show that an analytic function in a region with constant modulus is constant.
- 12 a Find the image of the circle $|z - 3i| = 3$ under the map $w = \frac{1}{z}$.
OR
b Derive the Jacobian of a transformation.
- 13 a Evaluate $\int_C |z| dz$ where C is the closed curve consisting of the upper semicircle $|z| = 1$ and the segment $-1 < x < 1$.
OR
b State and prove maximum modulus theorem.

Cont...

14 a State and prove Liouville's theorem.
OR

b Expand $\frac{1}{z(z-1)}$ as Laurent's series about $z = 0$ in powers of z .

15 a Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$

OR

Find the residue of $\frac{z}{z^2(z^2+q)}$ at its poles.

SECTION - C (30 Marks!

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

16 State and prove Cauchy - Riemann equations in polar form.

17 Let f be an analytic function defined in a region D . Let $Z_0 \in D$ if $f'(Z_0) \neq 0$ then show f is conformal at Z_0 .

18 State and prove Cauchy's integral formula.

19 State and prove Laurent's theorem.

20 Use residue theorem to evaluate $\int \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz$ around the circle $|z| = 2$.

Z-Z-Z

END