

PSG COLLEGE OF ARTS & SCIENCE  
(AUTONOMOUS)  
BSc DEGREE EXAMINATION MAY 2017  
(First Semester)

Branch- MATHEMATICS

CALCULUS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10x2 = 20)

If  $u = x^z + y^z + z^2$ ,  $x = e^t$ ,  $y = e^t \sin t$ ,  $z = e^t \cos t$ . Find  $\frac{du}{dt}$ .

State the necessary conditions for the existence of maxima or a minima of  $f(x,y)$ .

Write down the formula for finding the radius of curvature in Cartesian form.

Define evolute and involute of a curve.

State any two properties of definite integrals.

State Bemoullis formula.

Evaluate  $\int_0^1 \int_0^1 f(x^2+y^2) dy dx$ .

8 Evaluate  $\int_0^{\pi/2} \int_0^a r dr d\theta$ .

9 Find the value of  $\int_0^1 x^7(1-x)^8 dx$ .

10 Show that  $p(m,n) = p(n,m)$ .

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5x5 = 25)

11 a Find the maximum or minimum values of  $2(x^2 - y^2) - x^4 + y^4$ .

OR

b If  $z = f(x, y)$ ,  $x = r \cos \theta$ ,  $y = r \sin \theta$ , show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r} \frac{\partial z}{\partial r} + \frac{\partial^2 z}{\partial \theta^2}$$

12 a Prove that the radius of curvature at the point  $(a \cos \theta, a \sin \theta)$  on the

curve  $x^{2/3} + y^{2/3} = a^{2/3}$  is  $3a \sin \theta \cos \theta$ .

OR

b Find the envelope of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

13 a Evaluate  $\int \sec^{11} x dx$

OR

b Evaluate  $\int x^4 (\log x)^3 dx$

Evaluate  $\int_0^1 \int_0^1 e^{-y} dx dy$ .

OR

Evaluate  $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$ .

Derive the recurrence formula of Gamma functions.

OR

Evaluate  $\int_0^{\infty} e^{-x^2} dx$ .

SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

Find the shortest and longest distance from the point (1, 2, -1) to the sphere  $x^2 + y^2 + z^2 = 24$ .

Find the radius of curvature of the cardioid  $r = a(1 - \cos \theta)$

Prove that  $\int_0^{\pi/4} \log(1 + \tan \theta) d\theta = \frac{1}{2} \log 2$

Evaluate  $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\pi/2} \cos \theta \sin \phi \cos \psi d\theta d\phi d\psi$ .

Evaluate  $\int_0^a \int_0^{\sqrt{a^2 - x^2}} x^m y^n dx dy$  over the positive quadrant of the circle  $x^2 + y^2 = a^2$  in terms of Gamma functions, hence deduce area of the circle.