PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION IVfAY 2017
(Second Semester)

Branch- MATHEMATICS

ANALYTICAL GEOMETRY OF 3D & VECTOR CALCULUS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Markup
Answer ALL questions
. ALL questions carry EQUAL marks (10x2 = 20)

1 Find the equation of the sphere whose centre is (2, -3, 1) and radius is 5 units.

2 Write the condition that the plane Ax + By + Cz + D = 0 may touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

3 Prove that the equation $2x^2 + 2y^2 + 7Z^2$ - 1 Oyz - 1 Ozx + 2x + 2y + 26z -17 = 0 represents a cone whose vertex is (2, 2, 1).

4 Define the right circular cone.

5 Define a central quadrics.

6 Write the condition for the plane lx + my + nz = p is touch the conicoid $ax^2 + by^2 + cz^2 = 1$.

7 Find the unit vector normal to the surface $x^2 + 2y^2 + z^2 = 7$ at (1, -1, 2).

8 Prove that V. r = 3 and V x r = 6.

9 If $F = x^2i + y^2 j$, evaluate jF.dr along the line y = x from (0, 0) to (1, 1).

10 State the Green's theorem.

SECTION - B (25 Marks)
Answer ALL Questions
. ALL Questions Carry EQUAL Marks (5x5 = 25)

11 a Find the centre and radices of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

OR

b Show that the plane 2x - 2y + z + 12 = 0 touches the sphere $x^2 + y^2 + z^2$ - 2x - 4y + 2z = 3 and find the point of contact.

.12 a Show that the equation of a right circular cone whose vertex is 0, axis 0Z and semi-vertical angle a is $a^2 + y^2 = z^2 \tan 2a$.

OR

b Find the equations of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contains the line $\frac{X}{32} = \frac{y}{72} = \frac{z}{72}$.

13 a Find the equations of the tangent planes to $x^2 + y^2 + 4z^2 = 1$ which intersect in the line whose equations are 12x - 3y - 5 = 0, z = 1.

OR

b Derive the equation of any tangent plane to the coniciod $ax^2 + by^2 + cz^2 = 1$.

14 a Compute the divergence and curl of the vector
$F = xyzi + 3x^2yj + (xz^2 - y^2z)k$ at (1, 2,-1).

OR

b Find div curl F where $F = x^2yi + xzj + 2yzk$.

15 a' Evaluate jF.dr where $F = xyi + (x^2+y^2)j$ and Ci is the arc of the parabola $y = x^2 - 4$ from A(2, 0) to B(4, 12) in the XY plane.

OR

b Use divergence theorem to evaluate JjF. n ds where s

$F = x^J i + y \wedge j + k$ and s is the surface of the sphere $x^2 + y^2 + z^2$.

SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

16 Find the equation of the sphere passing through the points (1, 1, -1), (-5, 4, 2), (0, 2, 3) and having its centre on the.plane $3x + 4y + 2z = 6$.

17 Find the equation to the cone through the co-ordinate axes and the lines in which the plane $lx + my + nz = 0$ cuts the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$.

18 If OD is the diameter parallel to a secant APQ through A meeting the conicoid at P and Q, show that $\frac{APAQ}{OD^2}$ is constant.

19 Find the equation of the tangent plane and normal line to the surface xyz = 4 at the point i + 2j + 2k.

20 Verify Stoke's theorem for the function-$F = x^2i + xyj$ integrated round the square in the z = 0 plane whose sideo are along the lines x = 0, y = 0, x = a, y = a.

Z-Z-Z END