PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2017
(Sixth Semester)

Branch - MATHEMATICS

ALGEBRA-II

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks!
Answer ALL questions
ALL questions carry EQUAL marks (10x2 = 20)

1 Define a symmetric matrix and a skew symmetric matrix.

2 Prove that (iA)* =-iA*.

3 If V is a vector space over F prove that (i) a0 = 0 for a G F, (ii) Ov = 0 for v e V

4 Define a linearly dependent set in a vector space V over F.

5 Prove that A(W) is a subspace of $\hat{V}$.

6 Define an inner product space.

7 Find the rank of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

8 If 1, 2, 3 are the characteristic roots of the matrix A find the characteristics roots of A^1.

9 Define an algebra of linear transformation.

10 Define the range of T and rank of T for T€ A(V).

SECTION - B (25 Marks)
Answer ALL Questions
ALL Questions Carry EQUAL Marks (5x5 = 25)

11 a Show that every jhefmitian matrix H is uniquely expressible as A + iB, where A is real symmetric and B is real skew, symmetric.

OR

b Show that $U = \frac{1}{2}\begin{pmatrix} 1+i & -1+i \\ 1+i & 1-i \end{pmatrix}$ is an unitary matrix.

12 a If V is the internal direct sum of $U_1, U_2, \ldots\ldots U_n$, prove that V is isomorphic to the external direct sum of Ui, $U_2, \ldots\ldots U_n$.

OR

b If vi, $v_2, \ldots\ldots v_n$ are in V prove that either they are linearly independent or some v_k is a linear combination of the preceding ones vj,......... v_{k-1}.

13 a If V is finite-dimensional and v * 0 G V, prove that there is an element of G $\hat{V}$ such that f(v) * 0.

OR

b Let F be the real field and let V be the set of polynomials, in a variable x, over F of degree 2 or less. For p(x), q(x) G V, define an inner product by $\langle P(x), q(x) \rangle = \int_{-1}^{1} p(x)q(x)dx$. starting with the basis (1, x, x^2} obtain an ortho normal basis for V.

14 a Prove that the rank of the product of two matrices does not exceed the rank of either factor. •

OR -

b Verify the Cayley - Hamilton theorem for the matrix $A = \begin{vmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{vmatrix}$

• 15 a If V is finite - dimensional over F prove that Te A(V) is regular if and only if T maps V onto V.

OR ’ .

b If X e F is a characteristic root of T e A(V) prove that A. is a root of the minimal polynomial of T and in particular T only has a finite number of characteristic roots in F.

SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

' 16 Prove that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew symmetric matrix.

17 If V is finite dimensional and if W is a subspace of V, prove that W is finite dimensional, dim W < dim V and dim V/W = dim V - dim W.

18 If V and W are of dimensions m and n, respectively, over F, prove that Horn (V, W) is of dimensions mn over F.

19 Determine the characteristic vectors, and the characteristic subspace corresponding, to the rational characteristic roof of the matrix

i

1 -1 2

A = -2 1 3

3 2*.-3

20 If V is finite dimensional over F then prove that for S, T e A(V)
(i) r (ST) < r (T) (ii) r(TS) < r(T) "(iii) r(ST) = r(TS) = r(T) for S regular in A(V).

Z-Z-Z END

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