PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2017
(Fifth Semester).

Branch - MATHEMATICS

REAL ANALYSIS

Time : Three Hours **Maximum : 75 Marks**

SECTION-A (20 Marks)
Answer ALL questions
• ALL questions carry EQUAL marks (10x2 = 20)

1 Prove that every infinite subset of a countable set A is countable.
2 Prove that every neighbourhood is an open set.
3 Define finite subcover.
4 Define separated set.
5 Define complete set.

6 Suppose the radius of convergence of $\sum C_n Z^n$ is 1, and suppose $C_0 > C_1 > C_2 > \dots\dots\dots$, $\lim_{n\to\infty} C_n = 0$ Then prove that $\sum C_n Z^n$ converges at every point on the circle $|z| = 1$, except possibly at $z = 1$.

7 Consider $f(x) = \begin{cases} x + 2 & (-3 < x < -2) \\ -x - 2 & (-2 < x < 0) \\ x + 2 & (0 \le x < 1) \end{cases}$, Is it continuous at x = 0 or discontinuous at x = 0.

8 Define bounded set.
9 State mean value theorem

10 Let f be defined by f(x) = < x s*n ^ , Is f is differentiable at x
0 (x = 0)

SECTION - B (25 Marks)
Answer-ALL Questions
ALL Questions Carry EQUAL Marks (5x5 = 25)

11 a Let A be the set of all sequences whose elements are the digits 0 and 1. Then prove that this set A is uncountable.

OR

b Let $\{E_\alpha\}$ be a finite or infinite collection of sets E_α Prove that $\left(\bigcup_\alpha E_\alpha\right)^c = \bigcap_\alpha \left(E_\alpha^c\right)$

12 a Prove that compact subsets of metric spaces are closed.

OR

b Let P be a non-empty perfect set in R^k. Prove that P is uncountable.

13 a Suppose $\{S_n\}$ is monotonic. Prove that $\{S_n\}$ converges if and only if it is bounded.

OR

b $\sum \frac{1}{n^p}$ converges if p > 1 and diverges if p <1. Prove.

14 a Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

OR •

b If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, prove that f(E) is connected.

15 a State and prove chain rule for differentiation.

OR

b Suppose f is a real differentiable function on [a, b] and suppose

f'(a _ < A.'< f'(b) • Prove that there is a point x e (a,b) such that f'(x) = X.

SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3x10 = 30)

16 Prove that

(a) For any collection $\{G_a\}$ of open sets, U_a^a *ls* °P^en-

(b) For any collection $\{F_a\}$ of closed sets, fl_a^a *ls* dosed.

(c) For any finite collection Gi, G_2, ...G_n of open sets f|^n ^ Gj is open.

(d) For any finite collection Fj, F_2, ...F_n of closed sets U”$_{=1}$ $F_}$ is closed.

17 Prove that every K-cell is compact.

18 For any sequence $\{C_n\}$ of positive numbers, prove that

lim inf $_{n\to\infty}$ ^+t / C_n < lim inf $_{n\to\infty}$.

lim supn/c^ $_{n\to\infty}$ < lim sup $_{n\to\infty}$ n+t- Gfj

19 Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if f^{-1} (V) is open in X for every open set V in Y.

20 State and prove Taylor’s theorem.

Z-Z-Z END