PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2017
(Fifth Semester)

Branch - MATHEMATICS

ALGEBRA- I

Time: Three Hours . Maximum: 75 Marks

SECTION-A (20 Marks)
Answer ALL questions
ALL questions carry EQUAL marks (10x2 = 20)

1 Define commutative group with example.
2 Define left coset and right coset of H in G.
3 Define quotient group.
4 Define homomorphism of G into G, and prove that cp(e) = e, the unit element of G.
5 Let G be a group and cp an automorphism of G. Prove that if a e G is of order 0(a) > 0, then 0(cp(a) = 0(a).
6 If S has 9 elements then find the value (1 2 3) (5 6 4 1 8).
7 Define associative ring.
8 If U is an ideal of R and 1 G U, prove that U = R.
9 Let R be the ring'of all the real-valued, continuous functions on the closed unit interval. LetM = r^f(x)eR / f 2) = 0 >. Prove that M is a maximal ideal of R.
10 Let R be a Euclidean ring. Suppose that for a, b, c G R, a / be but (a, b) = 1. Prove that a / c.

SECTION - B (25 Marks)
Answer ALL Questions
ALL Questions Carry EQUAL Marks (5x5 = 25)

11 a If H is a non empty finite subset of a group G and H is closed under multiplication, then prove that H is a subgroup of G.

OR

b If G is a finite group whose order is a prime number P, then prove that G is a cyclic group.

12 a Define a normal subgroup of G. Prove that N is a normal subgroup of G if and only if gNg*1 = N for every g G G.

OR

b If <() is a homomorphism of G into G with Kernel K, then prove that K is a normal subgroup of G.

13 a Prove that if G is a group, then A(G), the set of automorphisms of G, is also a group.

OR

b Prove that every permutation is the product of its cycles.

14 a Prove that a finite integral domain is a field.

OR

b Define Kernel of cp. Prove that if cp is a homomorphism of R into R' with Kernel I(cp), then

(i) I(cp) is a subgroup of R under addition

(ii) If a G I(cp) and r G R then both ar and ra are in I(cp), Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Prove that R is a field.

OR

Let R be a Euclidean ring and let A be an ideal of R. Then prove that there exists an element a_0 e A such that A consists exactly of all a<)X as x ranges over R.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3x10 = 30)

Let G be the set of all 2 x 2 matrices a b V ,, c where a, b, c, d are real numbers, such that ad - be *0. Show that G is a group under matrix multiplication.

If H and K are finite subgroups of G of order 0(H) and O(K), respectively then prove that O(HK) = $\frac{0(H)G(K)}{O(HnK)}$.

State and prove Cayley's theorem.

Prove that if U is an ideal of the ring R, then R / U is a ring and is a homomorphic image of R.

Prove that the ideal A = (a_0) is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.

Z-Z-Z END