PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS & LAPLACE TRANSFORMS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

Find a function y=f(x) satisfying the differential equation $\frac{dy}{dx} = 1$ and the initial condition y(l)=5?

(i) $\frac{}{XX} - 6$ (ii) $\frac{}{} + 6$ (iii) $\frac{}{XX} + 6$ (iv) $\frac{1}{} - 6$

The necessary condition for the differential equation Mdx+Ndy=0 to be exact is

(i) $\frac{dN}{dx} \quad \frac{aM}{}$ (ii) $\frac{aN}{} \quad \frac{aM}{}$ (iii) $\frac{a^2M}{ay\,ay} \quad \frac{a^2N}{ax\,axay} \quad \frac{a^2N}{ay\,ax}$ (iv) $\frac{dM}{dy} \quad \frac{dN}{dx}$

3 Which one is a linearly dependent function?
(i) e & e^x (ii) cos x & sin x (iii)sin 2x & sin x cos x (iv) x & jxj

4 If $e^{'3x}$, cos 2x and sin 2x are all solutions of the differential equation $y^{lll}+3y^{11}+4y^{1}+12y=0$, then____ is also a solution of $y^{111}+3y^{u}+4y^{1}+12y=0$.
(i) Cie$^{'3x}$+C2Cos 2x+C3 sin 2x (ii) Cje$^{''3x}$ C2COS 2x C3 sin 2x
(iii) Cie$^{'3x}$/c2COS 2x C3 sin 2x (iv) cie$^{'3}$C2COS 2x/c3 sin 2x

The voltage drop in resistor is_____.

(i) I (ii) $\frac{TTQ}{C}$ (iii) $L\hat{} \frac{}{dt}$ (iv) RI

6 The general solution of y +b y=0 is_____.
(i) y(x)=cicos bx+C2 sin bx (ii) Cie^{x}+C2e$^{'x}$ (iii) ci+c_2x (iv) (ci+c_2)x

7 r(n + l) =_____.
(i) n! (ii) (n+1)! (iii) n (iv) $(n!)^2$

State the value of L $\frac{1}{3}\frac{i}{r}$ is j

(i) t^2 (ii) $2t^2$ (iii) $\frac{1}{2}r^{\frac{1}{2}}$ (iv) it

9 The convolution of $f(t)=e^{at}$ and $g(t)=e^{at}$ is
(i) e^{at} (ii) te^{at} (iii) $2e^{at}$ (iv) e^{at}/t

10 The transform of the triangular wave function is_____

(i) $\frac{1}{s}\tanh\frac{as}{2}$ (ii) tanh $\frac{as}{}$ ..... (iii) $\frac{1}{}-\tan\frac{as}{2}$ (iv) $\tanh\frac{}{2}$

SECTION - B (25 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Solve the differential equation $(6xy-y^3)dx+(4y-K3x^2-3xy^2)dy=0$.

OR

b Suppose that a time t=0, ten thousand people in a city with population M=100 thousand people have heard a certain rumor. After 1 week the number p(t) of those who have heard it has increased to p(l)=20 thousand. Assuming that p(t) satisfies a logistic equation, when will 80% of the city's population have heard the rumor?

12 Cont...

b Show that the functions yi(x)=e'3x, y_2(x)=cos 2x and y3(x)=sin 2x are linearly independent.

13 a Find a general solution of the fifth-order differential equation $9y^{(5)}-6y^{(4)}+y^{(3)}=0$.

OR

b Find a particular solution of $y^{11}+3y'+4y=3x+2$.

14 a Show that $L\{te^{at}\} = \frac{}{(s-a)^2}$ '

OR

b Find the inverse laplace transform of R(s) = $\frac{s^2+1}{s^3-2s^2-8s}$

15 a Find $L^{J}\{\tan ^J/)\}$.

OR

bi) Find L{g(t)} if g(t)= $\begin{matrix} 0 & \text{if} & t < 3, \\ *2 & \text{if} & t > 3. \end{matrix}$

ii) Find L{f(t)} iff(t)= $j^{C0\wedge 2t}$ **if $0 < t < 2n$**
if $t > 2TC.$

SECTION -C 140 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Solve the differential equation $2xy\frac{dy}{dx} = 4x^2+3y$.

OR

b Consider an animal population p(t) that is given by the equation

$\frac{dp}{dt} = 0.0004p^2 - 0.06p$. Find p(t) if (a)p(0)=200; (b) p(Q)=100.

17 a Show that the three solutions yi(x)=x, y_2(x)=x log x and y3(x)=x^2 of the third order equation.

$x^3y^{(3)}-x^2y^{(2)}+2xy^{(1)}-2y=0$ are linearly independent on the open interval x>0. Also find a particular solution that satisfies the initial conditions y(l)=3, $y^{(1)}$(l)=2, $y^{(2)}$(l)=l.

OR

b Discuss the application of linear differential equations.

18 a Solve the initial value problem $y^n-3y^!+2y=3e^{'x}-10\cos 3x$. y(0)=l, $y^1(0)=2$.

OR

b : Consider an RLC circuit with R=50Q, L=0.1H and c=5xl0'^{4}f. At time t=0, when both 1(0) and Q(0) are zero, the circuit is connected to a llOv, 60Hz alternating current generator. Find the current in the circuit and the time lag of the steady periodic current behind the voltage.

19 a Solve the initial value problem $x^n+4x=\sin 3t$, x(0)=x'(0)=0.

OR

b Consider the mass spring dashpot system with initial condition $x(0)=x^1(0)=0$ and with the imposed external force f(t)=15sin2t. Find the resulting transient motion and steady periodic motion of the mass.

20 a i) Find $L\{t^2 \sin kt\}$ ii) Find |.

OR

b Consider the RLC circuit with R=110Q, L=1H, C=0.001F and a battery supplying Eo^OOv. Initially there is no current in the circuit and no charge on the capacitor. At time t=0 the switch is closed and left closed for 1 second. At time t=l, it is opened and left open there after. Find the resulting current in the circuit.

Z-Z-Z END