

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

CALCULUS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 1 = 10)

- 1 The curvature of the curve is given by
(i) $|r'(0)|$ (ii) $|r'(r)|^3$ (iii) $r' \cdot r^{11}$ (iv) None of the above
- 2 Mention the correct value of $\nabla[u(t) \cdot v(t)]$
(i) $\frac{d}{dt}(w(0) \cdot v(0) + \dots)$ (ii) $M'(0)$ (iii) $v'(t)$ (iv) $u(t) \cdot v(t)$
- 3 Evaluate $f(3,2)$ where $f(x,y) =$
(i) 2 (ii) 3 (iii) 0 (iv) 1
- 4 If $f(x,y) = x^3 + x^2y^3 - 2y^2$, then find $f_x(2,1)$.
(i) 12 (ii) 14 (iii) 16 (iv) 5
- 5 Write $\frac{dy}{dx}$ for the implicit function $f(x,y) = 0$.
(i) $\frac{df}{dx}$ (ii) $\frac{\partial F}{\partial x}$ (iii) $\frac{dF}{dy}$ (iv) $-\frac{dF}{dx} \frac{dF}{dy}$
- 6 Define the directional derivative of $f(x,y)$ at (x_0, y_0) in the direction of a unit vector $w = \langle a, b \rangle$.
(i) ∇f (ii) u (iii) $\nabla f \cdot u$ (iv) $|\nabla f|$
- 7 Write down the relationship between Cartesian co-ordinate and polar co-ordinate,
(i) $x = r \cos \theta$ (ii) $y = r \sin \theta$ (iii) $x = r \cos Q$ (iv) $x = r \cos \theta, y = r \sin \theta$
- 8 Write down the value of $\int_0^a \int_0^b xy^2 dx dy$.
(i) $\frac{1}{3} ab^3$ (ii) 1 (iii) 4 (iv) 5
- 9 Find the value of $\int_0^a \int_0^b \int_0^c x^2 dy dz$.
(i) $\frac{1}{3} abc^2$ (ii) abc^2 (iii) abc (iv) abc^2
- 10 Write down the formula to convert from cylindrical to rectangular co-ordinate.
(i) (r, θ, z) to (x, y, z) (ii) $r dr d\theta dz = dx dy dz$ (iii) $dx dy dz$ (iv) None of the above

SECTION - B (25 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5x5 = 25)

- 11 a Find the vector equation and parametric equations for the line segment that joins the 2 points $(1,3,-2)$ and $(2,-1,3)$.
OR
- b Find (i) the derivative of $r(t) = (1 + t^3)i + te^t j + \sin 2tk$ and (ii) the unit tangent vector at the point $t=0$.
- 12 a Where is the function $h(x,y) = \arctan(\frac{y}{x})$ continuous?

13 a If $z=e^{x \sin y}$, where $x=st^2$ and $y=s^2t$, find $\frac{dz}{dr}$ and $\frac{dz}{dt}$.

OR

b Prove that (0,0) is one critical point of the function $f(x,y)=10x^2y-5x^2-4y^2-x^4-2y^4$

14 a Evaluate the integral $\int_0^{\pi} \int_0^1 \sin(y) dy dx$.

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OR

b Find the moment of inertia I_x, I_y and I_0 of a homogenous disk D with density $p(x,y) = P$, center the origin and radius "a".

15 a Find the cylindrical coordinates of the point with rectangular co-ordinates (3,-3,-7).

OR

b Convert spherical co-ordinates into rectangular co-ordinates.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5x8 = 40)

16 a Prove that the curvature of the curve given by the vector function is

$$K(t) = \frac{f(t)xf''(t)i}{|F'(t)|^3}$$

OR

b Find the curvature of the parabola $y=x^2$ at the points (0,0), (1,1) and (2,4).

17 a Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ if $x^3+y^3+z^3+6xyz=1$.

OR

b Show that $f(x,y)=x.e^{xy}$ is differentiable at (1,0) and find its linearization there.

18 a If $f(x,y,z)=x.\sin(yz)$

(i) Find the gradient of f and

(ii) Find the directional derivative of f at (1,3,0) in the direction of the vector $S = 1 + 2j - k$.

OR

b Find the equation of the tangent plane and normal line at the point (-2,1,3) to the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{3} = 3$.

19 a Evaluate $\iint_D xy dA$, where D is the region bounded by the line $y=x-1$ and the

parabola $y^2=2x+6$.

OR

b Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$ above the xy plane and inside the cylinder $x^2+y^2=2x$.

20 a A solid E lies within the cylinder $x^2+y^2=1$, below the plane $z=4$ and above the paraboloid $z=1-x^2-y^2$. the density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

OR

Evaluate $\iiint_B (x^2+y^2+z^2) dv$, where B is the unit ball

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