14MCU18

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2018
(Fifth Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

REAL ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (1 0 x 2 = 20)

1 Define an equivalence relation.

2 Give an example of a metric space.

3 State Weierstrass theorem.

4 Define connected sets.

5 Define subsequential limit.

6 Define complete metric space.

7 What do you mean by continuous functions?

8 Define discontinuity of the first kind.

9 Define local maximum at a point p ex.

10 State L'Hospital's rule.

SECTION- B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Let $\{E_n\}$, n = 1, 2, 3, ..., be a sequence of countable sets, and put

$$S - (jE_n_{n=l}^{oo}$$

. Then prove that S is countable.

OR

b Prove that every neighbourhood is an open set.

12 a Prove that compact subsets of compact sets are compact.

OR

b Let P be a nonempty perfect set in R^k. Then prove that P is uncountable.

13 a Prove $Vx^n_{n =0} = \frac{}{1 - x}$ if 0 < x < 1 and if x >1, the series diverge.

OR

Prove that e is irrational.

14 a If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X, then prove that f(E) is connected.

OR

b Prove that the continuous mapping of a compact set is compact.

15 a Let f be defined on [a, b]; if f has a local maximum at a point x e (a, b) and if fi(x) exists and then prove f (x) = 0.

<u>SECTION - C (30 Marks)</u>

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks **(3 x 10 = 30)**

16 If x is a metric space and $E \subset x$, then prove that (i) $\bar{E}$ is closed (ii) $E = \bar{E}$ iff E is closed (iii) $\bar{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$,

17 Prove that every k-cell is compact.

18 Prove $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

19 Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove f is uniformly continuous on X.

20 State and prove Taylor's theorem.

Z-Z-Z END