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PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2018

(Fifth Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

ABSTRACT ALGEBRA

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10x2 = 20)

1 Define order of an element of a group.
2 Define Normal Subgroup.
3 Define the Kernel of a homomorphism.
4 State Cayley’s theorem.
Define Zero-divisor.
6 State the pigeonhole principle.
7 Define maximal ideal.
8 Define principle ideal ring.
9 State necessary and sufficient conditions for two polynomials are to be equal.
10 Define irreducible polynomial.

SECTION - B (25 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

a Let G be the set of all 2x2 matrixes $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a,b,c,d are real numbers such that ad-bc^0. Check whether G is a group under matrix multiplication.

OR

b State and prove Lagrange’s theorem.

12 a If (j) is a homomorphism of G into G, then prove that

i) <J)(e) = e , the unit element of G ii) cj)(x—¹) = <j)(x)-¹ for all x € G.

OR

b Prove that every permutation can be uniquely expressed as a product of disjoint cycles.

13 a Define an associate ring.

OR

b If R is a ring, then for all a,b e R, prove that

i) a0=0a=0 ii) a(-b)=(-a)b=-(ab) iii) (-a) (-b)=ab iv) (-l)a=-a v) (-1)(-1)=1 where R has a unit element 1.

14 a If R is a commutative ring with unit element and M is an ideal of R, prove that M is a maximal ideal of R if and only if is a field.

OR

b State and prove Fermat’s theorem.

15 a State and prove the division algorithm.

OR

b If R is an integral domain, then so is R[x], Prove!.

SECTION - C f30 Marks)

Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 1 0 = 30)

Prove that the relation a = b mod H is an equivalence relation.

State and prove Cayley's theorem for abelian groups.

Discuss the Hamiltonian ring of real quaternions.

Prove that every integral domain can be imbedded in a field.

State and prove the Einstein criterion.

Z-Z-Z END