

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch - MATHEMATICS

DIFFERENTIAL EQUATIONS & LAPLACE TRANSFORMS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10x1 = 10)

Choose the correct answer:

1 An equation relating to an unknown function and one or more of its derivatives is called

- (i) Partial differential equations (ii) Differential equation
(iii) Exact equation (iv) None of these

The time rate of change of the temperature $T(t)$ of a body proportional to the difference between T and temperature A of the surrounding medium. This is

- (i) Triricell's law (ii) Newton's law of cooling
(iii) Mathematical model (iv) None of these

Wronskain of f and g is $W=$

- (i) $fg'+f'g$ (ii) $fg''+f''g$
(iii) $fg'-f'g$ (iv) None of these

4 The characteristic equation is $r^3+9r=0$. The complementary function is

- (i) c_1+c_2 as $3x+C_3 \sin 3x$ (ii) $c_1+c_2+c_3x^2$
(iii) $c_1e^{3x}+c_2e^{13x}+c_3e^x$ (iv) None of these

$y'' - tT(x)y' + (j)(x)y=f(x)$ has complementary function

- (i) $c_1y_1(x)+c_2y_2(x)$ (ii) $c_1e^x+c_2e^{2x}$
(iii) $C)\sin x+c_2\cos x$ (iv) None of these

Complementary function of $y''+4y=3x^j$ is

- (i) $C_1e^{2x}+c_2e^{ox}$ (ii) $c_1\cos 2x+ c_2\sin 2x$
(iii) $c_1+c_2xe^{2x}$ (iv) None of these

$L(e^{kt})$ is

- (i) $\frac{1}{s+k}$ (ii) $\frac{1}{s-k}$ (iii) $\frac{1}{s}$ (iv) None of these

$L^{-1}\left(\frac{1}{s^2}\right)$ is

- (i) 1 (ii) t (iii) t^2 (iv) t^3

The convolution is commutative if

- (i) $f+g=f'g$ (ii) $f*g=f-g$ (iii) $f'g=g*f$ (iv) None of these

10 $L\{tf(t)\}$ is

- (i) $F(s)$ (ii) $F''(s)$ (iii) $F(s)$ (iv) None of these

SECTION - B (25 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5x5 = 25)

11 a Show that $y(x)=2x^{i/j}-x^j$ In x satisfies the differential equation $4x^2y''+y=0$ for all $x>0$.

OR

12 a Let y_1 and y_2 be two solutions of the homogenous linear equation in $y^{(1)} + p(x)y' + q(x)y = 0$ on I . If C_1 and C_2 are constants, show that $y = C_1y_1 + C_2y_2$ is also a solution of $y^{(1)} + p(x)y' + q(x)y = 0$ on I .

OR

b Show that the functions $y_1(x) = e^{3x}$, $y_2(x) = \cos 2x$, $y_3(x) = \sin 2x$ are linearly independent.

13 a Calculate a particular solution of $y'' - 4y = 2e^{2x}$.

OR

b Solve the two dimensional system.

$$x' = -2y \quad y' = x$$

14 a Show that $L(t^a) = \frac{a!}{s^{a+1}}$

OR

b Calculate $L\{t \sin t\}$.

15 a Given $f(t) = \sin 2t$ and $g(t) = e^t$. Bring out the convolution of f and g .

OR

b Find $L\{g(t)\} * L\{f(t)\}$

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Find the solution of differential equation. $\frac{dy}{dx} = (x + y + 3)^2$

OR

b Discover the solution of $(6xy - y)dx + (4y + 3x - 3xy)dy = 0$.

17 a Discuss that three solutions $y_1(x) = x$, $y_2(x) = x/nx$ and $y_3(x) = x^2$ of third order equation.

$x^3y^{(3)} - x^2y^{(1)} + 2xy' - 2y = 0$ are linearly independent on the open interval $x > 0$. Also discuss the particular solution that satisfies $y(1) = 3$, $y'(1) = 2$, $y''(1) = 1$.

OR

b Discuss the solution the initial value problem.

$$y^{(3)} + 3y'' - 10y' = 0$$

$$y(0) = 7, \quad y'(0) = 0, \quad y''(0) = 70.$$

18 a Determine the appropriate form for a particular solution of the fifth-order equation.

$$(D-2)^3(D^2+9)y = x^2e^{2x} + x \sin 3x.$$

OR

b Consider an RLC circuit with $R = 50$ ohms (Ω), $L = 0.1$ henry(H), and $C = 5 \times 10^{-4}$ farad(F). At time $t = 0$ when both $I(0)$ and $Q(0)$ are zero, the circuit is connected to a 110-v, 60-Hz alternating current generator. Find the current in the circuit and the time lag of the steady periodic behind the voltage.

19 a Discover the inverse Laplace transform of $R(s) = \frac{s^2 + 1}{s^3 - 2s^2 - 8s}$.

OR

b Examine the initial value problem

$$x'' - x' - 6x = 0; \quad x(0) = 2, \quad x'(0) = -1.$$

20 a Discover $L\left(\frac{\sinh t}{t}\right)$.