

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2018
(Sixth Semester)

Branch - MATHEMATICS

COMPLEX ANALYSIS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Define a continuous function in the bounded closed domain D.
- 2 Show that $f(z) = z$ is nowhere differentiable.
- 3 Define Jacobian of a transformation.
- 4 Define inverse points with respect to a circle.

Evaluate $\int_C \frac{dz}{z}$ where C is circle $|z - 2| = 5$.

- 6 State Cauchy's fundamental theorem.
- 7 Define Primitive.
- 8 State fundamental theorem of algebra.
- 9 Find the residue of $\frac{f(z)}{z^e}$ at $z=0$.
- 10 Write Jordan's inequality.

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Prove that continuity is a necessary but not a sufficient condition for existence of a finite derivative.

OR

Show that the function $u = x^2 - 3xy^2$ is harmonic and find the corresponding analytic function.

- 12 a Prove that for the transformation $w = Jx^2 + y^2 - iy$, determine the region D^1 of the w-plane corresponding to the region D of the z-plane given circular disc $x^2 + y^2 < 1$.

OR

Prove that the superficial magnification of the conformal transformation $w=f(z)$ is $|f'(z)|^2$.

- 13 a Let $f(x)$ be continuous on a contour L of length 1 and let $|f(z)| < M$ on L, show that $|\int_L f(z) dz| < ML$.

OR

State and prove Cauchy's integral formula.

- 14 a Show that $0 < |z| < 4$, - $\sum_{n=0}^{\infty} \frac{z^{n+1}}{4^{n+1}}$

OR

b State and prove Maximum modulus principle.

- 15 a Find the residue for the function $f(z) = \frac{1}{(z^2+1)^3}$ at its poles.

SECTION - C (30 Marks)Answer any **THREE** Questions**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

- 16 Derive polar form of Cauchy-Riemann equation.
- 17 Find the image of the infinite strips $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = z^2$ show the region graphically.
- 18 State and prove Morera's theorem.
- 19 Find the Taylor's series to represent $\frac{1}{(z+2)(z+3)}$ in $|z| < 2$.
- 20 State and prove Cauchy's residue theorem.

Z-Z-Z

END